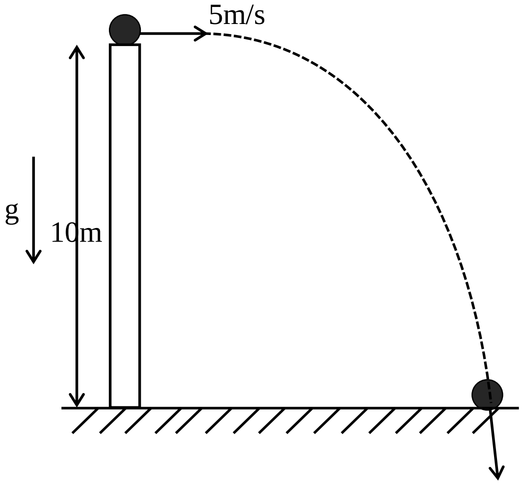
Physics

SECTION - A

1. A child stands on the edge of the cliff 10 m above the ground and throws a stone horizontally with an initial speed of 5 ms^{-1}. Neglecting the air resistance, the speed with which the stone hits the ground will be ___ ms^{-1} (given, $g = 10\ ms^{-2}$).

(1) 15 (2) 20 (3) 30 (4) 25

Sol. **(1)**



Along vertical direction

$u_y = 0$ $\quad$ $v_y^2 = u_y^2 + 2a_y g_y$

$a_y = +g$ $\quad$ $= (0)^2 + 2 \times 10 \times 10$

$v_y = ?$ $\quad$ $v_y^2 = 200$

s_y = 10 m

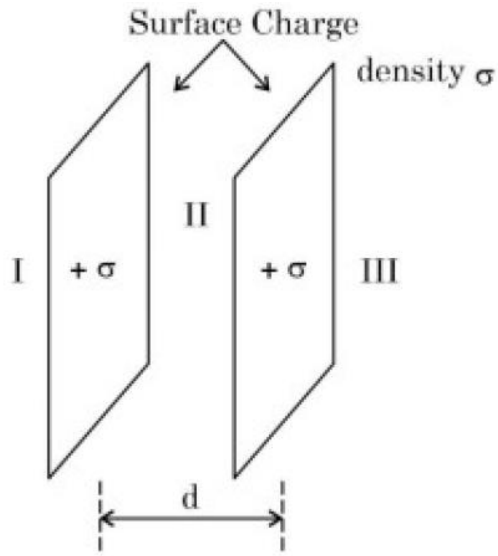
$\boxed{v_y^2 = 200}$

$\therefore \quad v = \sqrt{v_x^2 + v_y^2}$

$= \sqrt{25 + 200} \quad = \sqrt{225}$

= 15 m/s

2. Let σ be the uniform surface charge density of two infinite thin plane sheets shown in figure. Then the electric fields in three different region E_I, E_{II} and E_{III} are:



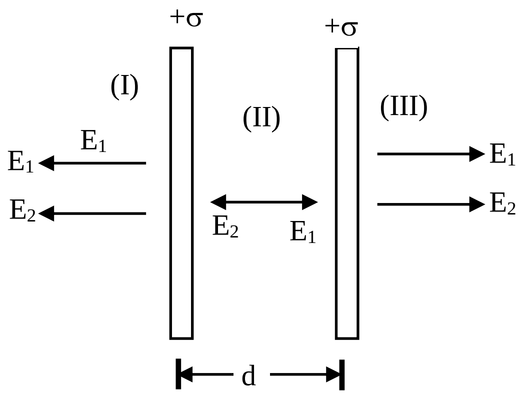
(1) $\vec{E}_I = \frac{2\sigma}{\epsilon_0}\hat{n}, \vec{E}_{II} = 0, \vec{E}_{III} = \frac{2\sigma}{\epsilon_0}\hat{n}$

(2) $\vec{E}_I = \frac{\sigma}{2\epsilon_0}\hat{n}, \vec{E}_{II} = 0, \vec{E}_{III} = \frac{\sigma}{2\epsilon_0}\hat{n}$

(3) $\vec{E}_I = -\frac{\sigma}{\epsilon_0}\hat{n}, \vec{E}_{II} = 0, \vec{E}_{III} = \frac{\sigma}{\epsilon_0}\hat{n}$

(4) $\vec{E}_I = 0, \vec{E}_{II} = \frac{\sigma}{\epsilon_0}\hat{n}, E_{III} = 0$

Sol. **(3)**



$$\therefore E_I = -\frac{\sigma}{E_0}\hat{n}$$

$$\therefore E_{II} = 0$$

$$\therefore E_{III} = -\frac{\sigma}{E_0}\hat{n}$$

3. A mercury drop of radius 10^{-3} m is broken into 125 equal size droplets. Surface tension of mercury is 0.45Nm^{-1}. The gain in surface energy is:

(1) 28×10^{-5} J (2) 17.5×10^{-5} J (3) 5×10^{-5} J (4) 2.26×10^{-5} J

Sol. **(4)**

[Volume of bigger drop] = [volume of smaller drop] × 125

$$\frac{4}{3}\pi R^3 = 125 \times \frac{4}{3}\pi r^3$$

$$R^3 = 125 r^3$$

$$\boxed{\therefore R = 5 \times r}$$

⇒ Gain in sinface energy = TdA

$$= 0.45 \times [A_2 - A_1]$$

$$= 0.45 \times [125 \times 4\pi r^2 - 4\pi R^2]$$

$$= 0.45 \times \left[125 \times 4\pi \left(\frac{R}{5}\right)^2 - 4\pi R^2\right]$$

$$= 0.45 \times [20\pi R^2 - 4\pi R^2]$$

$$= 0.45 \times 16\pi R^2$$

$$= 0.45 \times 16 \times 3.14 \times (10^{-3})^2$$

$$= 2.26 \times 10^{-5} \text{J}$$

4. If earth has a mass nine times and radius twice to that of a planet P. Then $\frac{v_e}{3}\sqrt{x}\ \text{ms}^{-1}$ will be the minimum velocity required by a rocket to pull out of gravitational force of P, where v_e is escape velocity on earth. The value of x is

(1) 1 (2) 3 (3) 18 (4) 2

Sol. **(4)**

$M_E = 9M_P$

$R_E = 2R_P$

$$V_c^1 = \sqrt{\frac{2GM_P}{R_P}} = \sqrt{\frac{2G\frac{M_E}{9}}{\frac{R_E}{2}}}$$

$$= \sqrt{\frac{2GM_E}{R_E}} \times \sqrt{\frac{2}{9}}$$

$$\boxed{V_c^1 = \frac{V_e}{3}\sqrt{2}}$$

5. A sample of gas at temperature T is adiabatically expanded to double its volume. The work done by the gas in the process is $\left(\text{given, } \gamma = \frac{3}{2}\right)$:

(1) $W = \frac{T}{R}[\sqrt{2} - 2]$ (2) $W = RT[2 - \sqrt{2}]$ (3) $W = TR[\sqrt{2} - 2]$ (4) $W = \frac{R}{T}[2 - \sqrt{2}]$

Sol. **(2)**

Work done in the process is given by

$$W = \frac{R}{\gamma - 1}(T_1 - T_2)$$

For adiabatic process:

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$TV^{\frac{3}{2}-1} = T_2(2V)^{\frac{3}{2}-1}$$

$$TV^{\frac{1}{2}} = T_2(2V)^{\frac{1}{2}}$$

$$T^2V = T_2^2 \times 2V$$

$$\therefore T_2 = \frac{T}{\sqrt{2}}$$

$$\therefore W = \frac{R}{\gamma - 1} \times \left(T - \frac{T}{\sqrt{2}}\right)$$

$$= 2RT\left[1 - \frac{1}{\sqrt{2}}\right]$$

$$= RT\left[2 - \frac{2}{\sqrt{2}}\right]$$

$$= RT[2 - \sqrt{2}]$$

$$\boxed{W = RT\left[2 - \sqrt{2}\right]}$$

6. $\left(P+\frac{a}{V^2}\right)(V-b)=RT$ represents the equation of state of some gases. Where P is the pressure, V is the volume, T is the temperature and a, b, R are the constants. The physical quantity, which has dimensional formula as that of $\frac{b^2}{a}$, will be:

(1) Compressibility
(2) Energy density
(3) Modulus of rigidity
(4) Bulk modulus

Sol. **(1)**

$[b]=[L^3]$

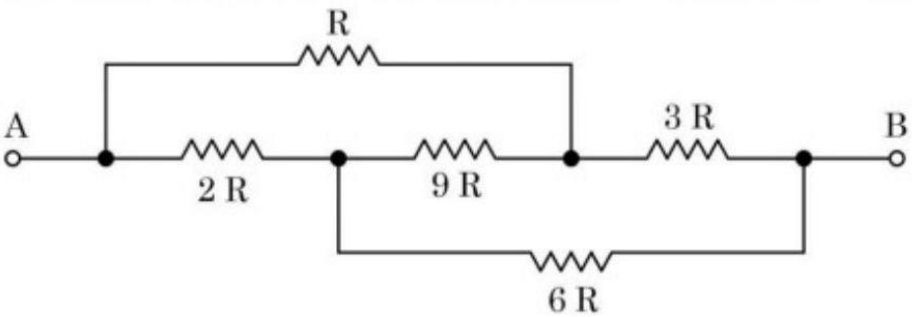
$[a]=[PV^2]$

$=[ML^{-1}T^{-2}][L^6]$

$=[ML^5T^{-2}]$

$$\frac{[b^2]}{[a]}=\frac{[L^6]}{[ML^5T^{-2}]}=[M^{-1}L^1T^2]$$

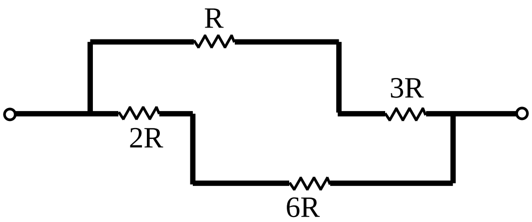
7. The equivalent resistance between A and B of the network shown in figure:



(1) $\frac{8}{3}R$
(2) 21R
(3) 14R
(4) $11\frac{2}{3}R$

Sol. **(1)**

∵ The given network is wheat-stone network



$$\therefore \quad R_{eq}=\frac{4R\times 8R}{4R+8R}$$

$$=\frac{4R\times 8R}{12R}$$

$$\boxed{R_{eq}=\frac{8}{3}R}$$

8. Match List I with List II:

List I	List II
A. AC generator	I. Presence of both L and C
B. Transformer	II. Electromagnetic Induction
C. Resonance phenomenon to occur	III. Quality factor
D. Sharpness of resonance	IV. Mutual Induction

Choose the correct answer from the options given below:

(1) A-IV, B-III, C-I, D-II
(2) A-IV, B-II, C-I, D-III
(3) A-II, B-IV, C-I, D-III
(4) A-II, B-I, C-III, D-IV

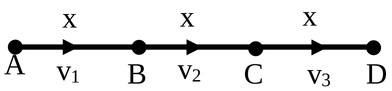
Sol. **(3)**

(A) A.C. generator $\rightarrow$ II. Electro-magnetic induction
(B) transformer $\rightarrow$ IV Mutual induction
(C) Resonance phenomenon to occur $\rightarrow$ (I) presence of both L and C
(D) Sharpness of resonance $\rightarrow$ (III) Quality factor

9. An object moves with speed v_1, v_2 and v_3 along a line segment AB, BC and CD respectively as shown in figure. Where $AB = BC$ and $AD = 3AB$, then average speed of the object will be:

(1) $\frac{(v_1+v_2+v_3)}{3v_1v_2v_3}$
(2) $\frac{(v_1+v_2+v_3)}{3}$
(3) $\frac{3v_1v_2v_3}{(v_1v_2+v_2v_3+v_3v_1)}$
(4) $\frac{v_1v_2v_3}{3(v_1v_2+v_2v_3+v_3v_1)}$

Sol. **(3)**



$$\langle v \rangle = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{3x}{\frac{x}{v_1} + \frac{x}{v_2} + \frac{x}{v_3}}$$

$$= \frac{3}{\left[\frac{1}{v_1} + \frac{1}{v_2} + \frac{1}{v_3}\right]} = \frac{3}{\left[\frac{v_2v_3 + v_1v_3 + v_1v_2}{v_1v_2v_3}\right]}$$

$$\langle v \rangle = \frac{3v_1v_2v_3}{[v_3v_2 + v_1v_2 + v_1v_2]}$$

10. 'n' polarizing sheets are arranged such that each makes an angle 45° with the preceeding sheet. An unpolarized light of intensity I is incident into this arrangement. The output intensity is found to be $I/64$. The value of n will be:

(1) 4 (2) 3 (3) 5 (4) 6

Sol. **(D)**

According to Malus law:

$$I = \frac{I_0}{2}\left[\cos^2 45 \times \cos^2 45 \times \cos^2 45 \times ...(n-1)\,\text{times}\right]$$

$$\frac{I_0}{64} = \frac{I_0}{2} \times \left(\frac{1}{2}\right)^{n-1}$$

$$\frac{1}{32} = \frac{1}{2^{n-1}} \Rightarrow \frac{1}{(2)^5} = \frac{1}{2^{n-1}}$$

$\therefore$ n – 1 = 5

$\boxed{\therefore n = 6}$

11. Match List I with List II:

List I	List II
A. Microwaves	I. Radio active decay of the nucleus
B. Gamma rays	II. Rapid acceleration and deceleration of electron in aerials
C. Radio waves	III. Inner shell electrons
D. X-rays	IV. Klystron valve

Choose the correct answer from the options given below:

(1) A-I, B-III, C-IV, D-II (2) A-IV, B-I, C-II, D-III

(3) A-IV, B-III, C-II, D-I (4) A-I, B-II, C-III, D-IV

Sol. **(B)**

(A) Micro-wave (IV) Klystron valve

(B) Gamma rays (I) Radio-active decay of nucleus

(C) Radio-waves (II) Rapid acceleration and deceleration of electrons in aerials

(D) X-rays (III) Inner shell electron

12. A proton moving with one tenth of velocity of light has a certain de Broglie wavelength of λ. An alpha particle having certain kinetic energy has the same de-Brogle wavelength λ. The ratio of kinetic energy of proton and that of alpha particle is:

(1) 2: 1 (2) 1: 2 (3) 1: 4 (4) 4: 1

Sol. **(C)**

The wavelength of matter is given by

$$\boxed{\lambda = \frac{h}{p}}$$

$$\frac{\lambda_p}{\lambda_\alpha} = \frac{p_\alpha}{p_p} = \frac{\sqrt{2k_\alpha m_\alpha}}{\sqrt{2k_p m_p}} = 1$$

$$\therefore \frac{k_\alpha}{k_p} \times \frac{m_\alpha}{m_p} = 1 \Rightarrow \frac{k_\alpha}{k_p} = \frac{m_p}{m_\alpha}$$

$$\boxed{\frac{k_\alpha}{k_p} = \frac{1}{4}}$$

13. A block of mass 5 kg is placed at rest on a table of rough surface. Now, if a force of 30 N is applied in the direction parallel to surface of the table, the block slides through a distance of 50 m in an interval of time 10 s. Coefficient of kinetic friction is (given, $g = 10\ ms^{-2}$):

(1) 0.60 (2) 0.25 (3) 0.75 (4) 0.50

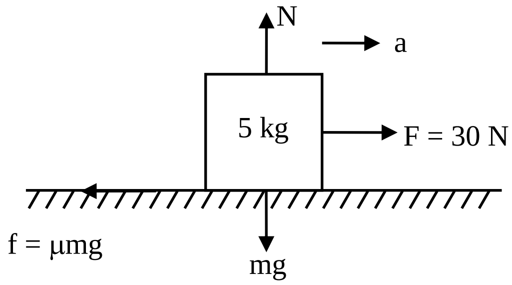
Sol. **(D)**

$$S = ut + \frac{1}{2}at^2$$

$$50 = 0 \times t + \frac{1}{2} \times a \times (10)^2$$

$$50 = \frac{1}{2} \times a \times 100$$

$$a = \frac{100}{100} \Rightarrow \boxed{a = 1\ m/s^2}$$



$$\sum F_x = ma_x$$

$$30 - \mu mg = ma$$

$$30 - \mu \times 50 = 5$$

$$50\mu = 25$$

$$\mu = \frac{25}{50}$$

$$= \frac{1}{2}$$

$$\Rightarrow \boxed{\mu = 0.5}$$

14. Given below are two statements:

Statement I: Acceleration due to gravity is different at different places on the surface of earth.

Statement II: Acceleration due to gravity increases as we go down below the earth's surface.

In the light of the above statements, choose the correct answer from the options given below

(1) Statement I is false but Statement II is true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are false

(4) Both Statement I and Statement II are true

Sol. **(B)**

Statement (I) is true But

Statement (II) is false

15. Which of the following frequencies does not belong to FM broadcast.

(1) 64MHz (2) 89MHz (3) 99MHz (4) 106MHz

Sol. **(A)**

The Frequencies for FM Broadcast is between 87.5 MHz to 108 MHz.

16. The mass of proton, neutron and helium nucleus are respectively $1.0073u$, $1.0087u$ and $4.0015u$. The binding energy of helium nucleus is:

(1) 28.4MeV (2) 56.8MeV (3) 14.2MeV (4) 7.1MeV

Sol. **(A)**

$2P + 2n = {}^{4}_{2}He + E$

$\therefore$ B.E = [2 × (1.0073 + 1.0087) – 4.0015] × 931

= 0.0305 × 931

= 28.3955 MeV

17. A steel wire with mass per unit length 7.0×10^{-3} kg m^{-1} is under tension of 70 N. The speed of transverse waves in the wire will be:

(1) 100 m/s (2) 10 m/s (3) 50 m/s (4) 200πm/s

Sol. **(A)**

The velocity of Transverse wave on string is given by

$$V = \sqrt{\frac{T}{\mu}}$$

$$= \sqrt{\frac{70}{7 \times 10^{-3}}} = \sqrt{\frac{70 \times 10^{3}}{7}}$$

$$= \sqrt{10^4} = 100 \text{ m/s}$$

18. Match List I with List II:

List I	List II
A. Intrinsic semiconductor	I. Fermi-level near the valence band
B. n-type semiconductor	II. . Fermi-level in the middle of valence and conduction band
C. p-type semiconductor	III. Fermi-level near the conduction band
D. Metals	IV. Fermi-level inside the conduction band

Choose the correct answer from the options given below:

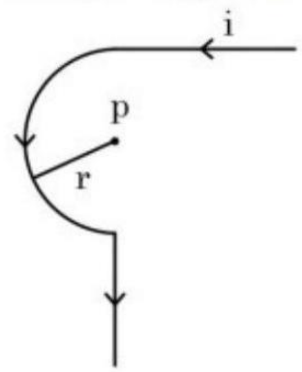
(1) A-II, B-III, C-I, D-IV (2) A-I, B-II, C-III, D-IV

(3) A-II, B-I, C-III, D-IV (4) A-III, B-I, C-II, D-IV

Sol. **(A)**

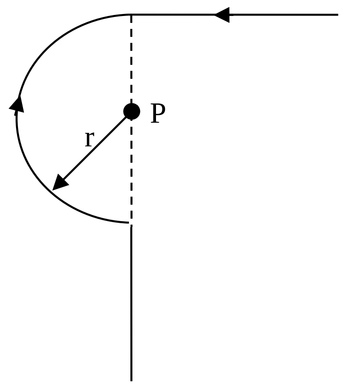
(A) Intrinsic	(II) Fermi-level in the middle of valence and conduction band
(B) n-type semiconductor	(III) Fermi-level near conduction band
(C) p-type semiconductor	(I) Fermi-level near valence band
(D) Metals	(IV) Fermi-level inside the conduction band

19. Find the magnetic field at the point P in figure. The curved portion is a semicircle connected to two long straight wires.



(1) $\frac{\mu_0 i}{2r}\left(1+\frac{2}{\pi}\right)$ (2) $\frac{\mu_0 i}{2r}\left(\frac{1}{2}+\frac{1}{2\pi}\right)$ (3) $\frac{\mu_0 i}{2r}\left(1+\frac{1}{\pi}\right)$ (4) $\frac{\mu_0 i}{2r}\left(\frac{1}{2}+\frac{1}{\pi}\right)$

Sol. **(B)**



$B_P = B_1 + B_2$

$$= \frac{\mu_0 i}{4\pi r} + \frac{\mu_0 i}{4r}$$

$$= \frac{\mu_0 i}{4r}\left[\frac{1}{\pi}+1\right]$$

$$\boxed{B_P = \frac{\mu_0 i}{2r}\left[\frac{1}{2\pi}+\frac{1}{2}\right]}$$

20. The average kinetic energy of a molecule of the gas is

(1) proportional to absolute temperature (2) proportional to pressure
(3) proportional to volume (4) dependent on the nature of the gas

Sol. **(A)**

The average kinetic energy of gas molecule is given by,

$$K.E_{avg} = \frac{3}{2}KT$$

$$\boxed{\therefore K.E_{avg} \propto T}$$

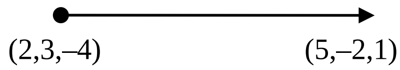
SECTION - B

21. A small particle moves to position $5\hat{\imath} - 2\hat{\jmath} + \hat{k}$ from its initial position $2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$ under the action of force $5\hat{\imath} + 2\hat{\jmath} + 7\hat{k}$ N. The value of work done will be _________ J.

Sol. **40**

$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$

$= \left(5\hat{i} - 2\hat{j} + \hat{k}\right) - \left(2\hat{i} + 3\hat{j} - 4\hat{k}\right)$



$\overrightarrow{\Delta r} = 3\hat{i} - 5\hat{j} + 5\hat{k}$

$\therefore$ W = $\vec{F}\cdot\overrightarrow{\Delta r}$

$= \left(5\hat{i} + 2\hat{j} + 7\hat{k}\right)\cdot\left(3\hat{i} - 5\hat{j} + 5\hat{k}\right)$

= 15 – 10 + 35

= 5 + 35

$\boxed{W = 40J}$

22. A certain pressure 'P' is applied to 1 litre of water and 2 litre of a liquid separately. Water gets compressed to 0.01% whereas the liquid gets compressed to 0.03%. The ratio of Bulk modulus of water to that of the liquid is $\frac{3}{x}$.

The value of x is _________.

Sol. **1**

Bulk Modulus = $V\dfrac{dP}{dV}$

$$\frac{(B)_{water}}{(B)_{liquid}} = \frac{V\ dP/dV}{V\ dP/dV} = \frac{dP/0.01}{dP/0.03}$$

$$\therefore \frac{(B)_{water}}{(B)_{liquid}} = \frac{0.03}{0.01} = \frac{3}{1}$$

$$\boxed{\frac{(B)_{water}}{(B)_{liquid}} = \frac{3}{1}}$$

$\therefore$ On comparing with $\dfrac{3}{x}$, The value of "x" will be "1'.

23. A light of energy 12.75eV is incident on a hydrogen atom in its ground state. The atom absorbs the radiation and reaches to one of its excited states. The angular momentum of the atom in the excited state is $\frac{x}{\pi} \times 10^{-17}$eVs. The value of x is __________ (use $h = 4.14 \times 10^{-15}$eVs, $c = 3 \times 10^{8}$ ms^{-1}).

Sol. **x = 828**

The energy of electron in ground state = –13.6 eV

$E_n - E_1 = 12.75$

$\therefore E_n = 12.75 - 13.6$

$E_n = -0.85$

So "n" is given by

$$E_n = -\frac{13.6}{n^2}$$

$$n^2 = \frac{-13.6}{-0.85}$$

$n^2 = 16 \Rightarrow \boxed{n = 4}$

$\Rightarrow L = \frac{nh}{2\pi} = \frac{x}{\pi} \times 10^{-17}$

$\Rightarrow 4 \times \frac{h}{2\pi} = \frac{x}{\pi} \times 10^{-17}$

$4 \times \frac{4.14 \times 10^{-15}}{2\pi} = \frac{x}{\pi} \times 10^{-17} \Rightarrow \frac{2 \times 4.14 \times 10^{-15}}{10^{-17}} = x$

$x = 8.28 \times 10^2 \Rightarrow \boxed{x = 828}$

24. A charge particle of $2\mu C$ accelerated by a potential difference of 100 V enters a region of uniform magnetic field of magnitude 4mT at right angle to the direction of field. The charge particle completes semicircle of radius 3 cm inside magnetic field. The mass of the charge particle is _______ $\times 10^{-18}$ kg.

Sol. **144**

$R = \frac{mv}{qB} = \frac{p}{qB}$

$R = \frac{\sqrt{2mq\Delta V}}{qB}$

$3 \times 10^{-2} = \frac{\sqrt{2m \times 2 \times 10^{-6} \times 10^2}}{2 \times 10^{-6} \times 4 \times 10^{-3}}$

$3 \times 10^{-2} \times 2 \times 10^{-6} \times 4 \times 10^{-3} = \sqrt{4m \times 10^{-4}}$

$24 \times 10^{-11} = \sqrt{4m \times 10^{-4}}$

$m = \frac{24 \times 24 \times 10^{-22}}{4 \times 10^{-4}}$

$\boxed{m = 144 \times 10^{-18}\,\text{Kg}}$

25. The amplitude of a particle executing SHM is 3 cm. The displacement at which its kinetic energy will be 25% more than the potential energy is: _________ cm.

Sol. **2**

$K.E = P.E + \frac{25}{100} \times P.E.$

$K.E = P.E + \frac{1}{4} P.E$

$K.E = \frac{5}{4} P.E$

$\frac{1}{2} K\left(A^2 - x^2\right) = \frac{5}{4} \times \frac{1}{2} Kx^2$

$4(A^2 - x^2) = 5x^2$

$4A^2 - 4x^2 = 5x^2$

$9x^2 = 4A^2$

$x^2 = \frac{4}{9} \times (3)^2$

$\boxed{\therefore x = \pm 2}$

26. In an experiment to find emf of a cell using potentiometer, the length of null point for a cell of emf 1.5 V is found to be 60 cm. If this cell is replaced by another cell of emf E, the length-of null point increases by 40 cm. The value of E is $\frac{x}{10}V$. The value of x is ________.

Sol. **25**

$E_1 = K\ell_1$ ...(i)

$E_2 = K\ell_2$ ...(ii)

$\therefore \frac{E_2}{E_1} = \frac{\ell_2}{\ell_1}$

$\frac{E}{1.5} = \frac{100}{60}$

$\therefore E = 1.5 \times \frac{10}{6}$

$= \frac{3}{2} \times \frac{10}{6}$

$= \frac{5}{2}$

$= 2.5$

$= \frac{25}{10}$

$\boxed{\therefore x = 25}$

27. A thin cylindrical rod of length 10 cm is placed horizontally on the principle axis of a concave mirror of focal length 20 cm. The rod is placed in a such a way that mid point of the rod is at 40 cm from the pole of mirror. The length of the image formed by the mirror will be $\frac{x}{3}$ cm. The value of x is ________.

Sol. **32**

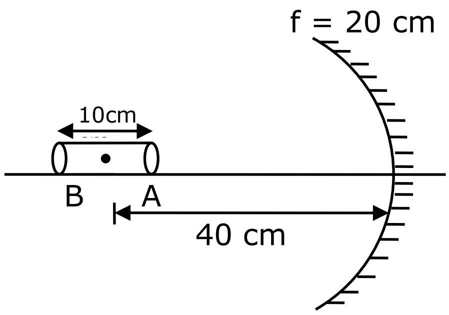


Image of end A:
u = –35 cm
f = –20 cm
v = ?

$v = \frac{uf}{u - f}$

$= \frac{-35 \times -20}{-35 + 20}$

$= \frac{-35 \times -20}{-15}$

$$\boxed{v = -\frac{140}{3}}$$

Image of end B:

u = –45 cm

v = ?

f = – 20 cm

$$v = \frac{uf}{u-f}$$

$$= \frac{-45 \times -20}{-45+20}$$

$$= \frac{-45 \times -20}{-25}$$

v = −36

$$\therefore \text{ length of image } = \left| -36 + \frac{140}{3} \right|$$

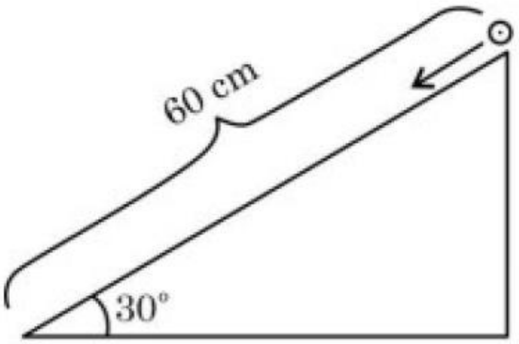
$$= \left| -\frac{108+140}{3} \right|$$

$$= \frac{32}{3}$$

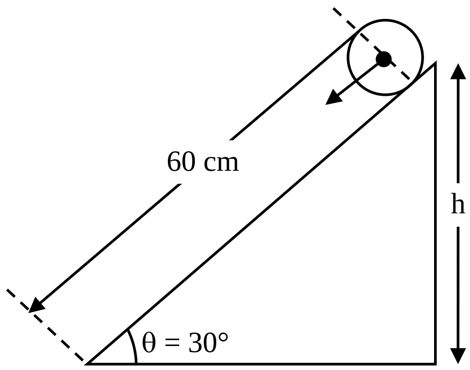
$\therefore$ The value of x = 32

28. A solid cylinder is released from rest from the top of an inclined plane of inclination 30° and length 60 cm. If the cylinder rolls without slipping, its speed upon reaching the bottom of the inclined plane is __________ ms^{-1}.

(Given g = 10 ms^{-2})



Sol.



h = 60 sin 30

$\boxed{\therefore\ h = 30 \text{ cm}}$

The velocity of by linder upon reaching the ground is given by

$$V = \sqrt{\frac{2gh}{1+\frac{K^2}{R^2}}}$$

$$\therefore V = \sqrt{\frac{2\times 10\times 30\times 10^{-2}}{1+\frac{1}{2}}}$$

$$= \sqrt{\frac{6\times 2}{3}}$$

$$\boxed{V = 2\ \text{m/s}}$$

29. A series LCR circuit is connected to an ac source of $220\ \text{V}, 50\ \text{Hz}$. The circuit contain a resistance $R = 100\Omega$ and an inductor of inductive reactance $X_L = 79.6\Omega$. The capacitance of the capacitor needed to maximize the average rate at which energy is supplied will be _________ μF.

Sol. **40**

For maximum power, the LCR must be in resonance.

$\therefore X_L = X_C$

$$79.6 = \frac{1}{\omega C}$$

$$C = \frac{1}{\omega\times 79.6}$$

$$= \frac{1}{2\pi\times 50\times 79.6}$$

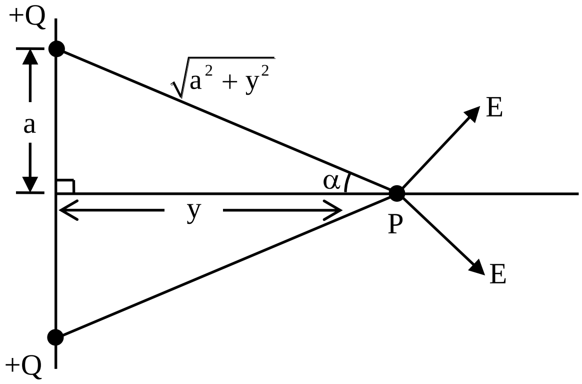
$$= \frac{1}{100\pi\times 79.6}$$

$= 40 \times 10^{-6}$

$$\boxed{C = 40\mu F}$$

30. Two equal positive point charges are separated by a distance $2a$. The distance of a point from the centre of the line joining two charges on the equatorial line (perpendicular bisector) at which force experienced by a test charge q_0 becomes maximum is $\frac{a}{\sqrt{x}}$. The value of x is _________.

Sol. **2**



Electric field at point "P" due to any one change = $\frac{KQ}{a^2+y^2}$

$\therefore$ Net electric field at point "P" will be

$E_{net} = 2E \cos \alpha$

$= \frac{2KQ}{a^2+y^2} \times \frac{y}{\sqrt{a^2+y^2}}$

$E_{net} = \frac{2KQy}{(a^2+y^2)^{3/2}}$

$\Rightarrow$ Electric force (F) = $E_{net}\, q_0$

$= \frac{2K\,Qq_0 y}{\left(a^2+y^2\right)^{3/2}}$

For F = max $\Rightarrow \frac{dF}{dy} = 0$

By solving, we get $\boxed{y = \frac{a}{\sqrt{2}}}$

$\therefore$ the value of $\boxed{x = 2}$

Chemistry

SECTION - A

31. A solution of $FeCl_3$ when treated with $K_4[Fe(CN)_6]$ gives a prussiun blue precipitate due to the formation of

(1) $K[Fe2(CN)_6]$(2) $Fe_4[Fe(CN)_6]_3$(3) $Fe[Fe(CN)_6]$(4) $Fe_3[Fe(CN)_6]_2$

Sol. **2**

$4Fecl_3 + 3K_4\,[Fe(CN)_6] \longrightarrow 12KCl + Fe_4[Fe(CN)_6]_3$

Pursianblue ppt

32. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason **R**

Assertion A: Hydrogen is an environment friendly fuel.

Reason R: Atomic number of hydrogen is 1 and it is a very light element.

In the light of the above statements, choose the correct answer from the options given below

(1) A is true but **R** is false

(2) **A** is false but **R** is true

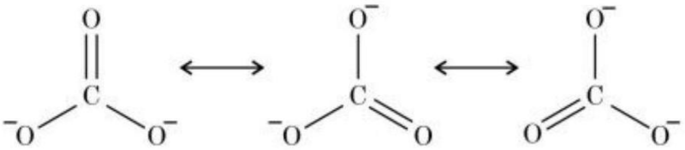
(3) Both **A** and **R** are true and **R** is the correct explanation of **A**

(4) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

Sol. **4**

No pollution occurs by combustion of hydrogen and very low density of hydrogen.

33. Resonance in carbonate ion (CO_3^{2-}) is



Which of the following is true?

(1) All these structures are in dynamic equilibrium with each other.

(2) It is possible to identify each structure individually by some physical or chemical method.

(3) Each structure exists for equal amount of time.

(4) $CO_3{}^{2-}$ has a single structure i.e., resonance hybrid of the above three structures.

Sol. **4**

Resonating structure are hypothtical and resonance hybrid is a real structure which is weighted average of all the resonating struture.

34. Match List I with List II

List I		List II
(A)	Tranquilizers	(I) Anti blood clotting
(B)	Aspirin	(II) Salvarsan
(C)	Antibiotic	(III) antidepressant drugs
(D)	Antiseptic	(IV) soframicine

Choose the correct answer from the options given below:

(1) (A) –IV, (B) – II, (C) – I, (D) – III(2) (A) – II, (B) – I, (C) – III, (D) – IV

(3) (A) - III, (B) - I, (C) - II, (D) – IV (4) (A) - II, (B) –IV, (C) – I, (D) – III

Sol. **3**

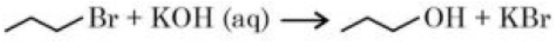
A $\rightarrow$ (iii)

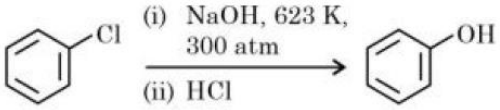
B $\rightarrow$ (i)

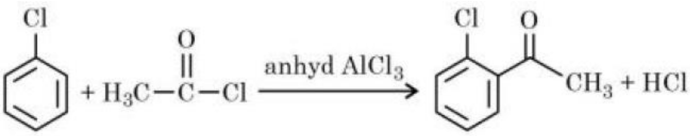
C $\rightarrow$ (ii)

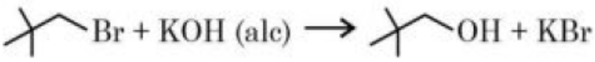
D $\rightarrow$ (iv)

35. Identify the incorrect option from the following:

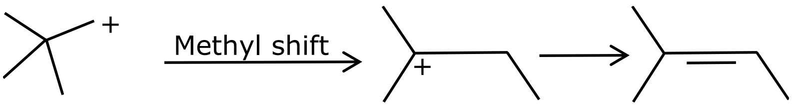
(1) 

(2) 

(3) 

(4) 

Sol. **4**



In question given option reaction is incorrect so right answer is (4)

36. But-2-yne is reacted separately with one mole of Hydrogen as shown below:

$$\underline{B} \xleftarrow[\text{liq } NH_3]{Na} \underset{+H_2}{CH_3-C\equiv C-CH_3} \xrightarrow[\Delta]{Pd/C} \underline{A}$$

A. A is more soluble than B.

B. The boiling point & melting point of A are higher and lower than B respectively.

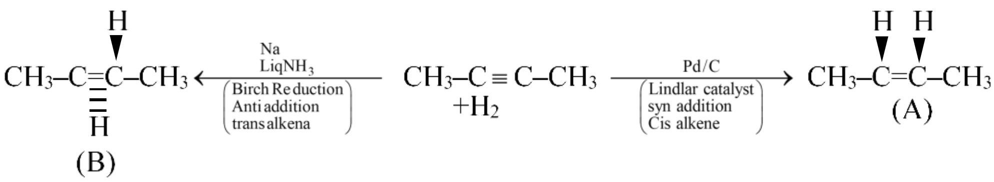
C. A is more polar than B because dipole moment of A is zero.

D. Br_2 adds easily to B than A.

Identify the incorrect statements from the options given below:

(1) B, C & D only (2) A and B only (3) A, C & D only (4) B and C only

Sol. **2**



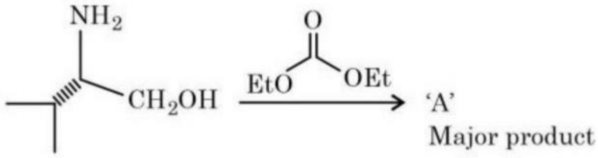
A) Cis has dipole monent, more soluble than trans (B)

B) B.P.(cis > trans), M.P. (trans > cis)

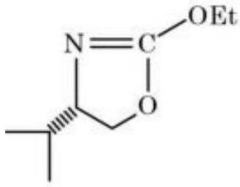
C) Dipole moment (A > B) but $\mu_A \neq 0$

D) Br_2 add easily to A not B

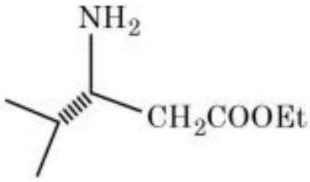
37. In the following reaction, ' A ' is



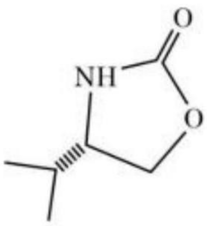
(1)



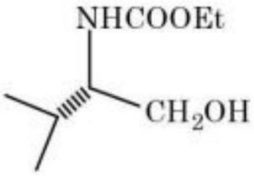
(2)



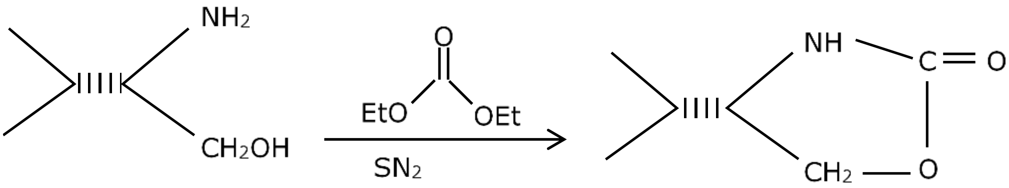
(3)



(4)



Sol. **3**



1

38. Highest oxidation state of Mn is exhibited in Mn_2O_7. The correct statements about Mn_2O_7 are

(A) Mn is tetrahedrally surrounded by oxygen atoms.

(B) Mn is octahedrally surrounded by oxygen atoms.

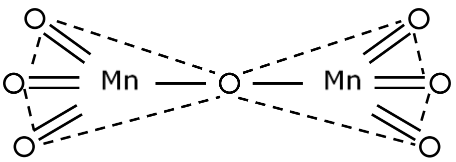
(C) Contains Mn-O-Mn bridge.

(D) Contains Mn-Mn bond.

Choose the correct answer from the options given below:

(1) A and Conly (2) A and D only (3) B and C only (4) B and D only

Sol. **1 (A & C)**



39. Match List I with List II

	List I	List II
(A)	Slaked lime	(I) $NaOH$
(B)	Dead burnt plaster	(II) $Ca(OH)_2$
(C)	Caustic soda	(III) $Na_2CO_3 \cdot 10H_2O$
(D)	Washing soda	(IV) $CaSO_4$

Choose the correct answer from the options given below:

(1) (A) - III, (B) - IV, (C) - II, (D) – I

(2) (A) - III, (B) - II, (C) - IV, (D) – I

(3) (A) - I, (B) - IV, (C) - II, (D) – III

(4) (A) –II, (B) – IV, (C) - I, (D) - III

Sol. **4**

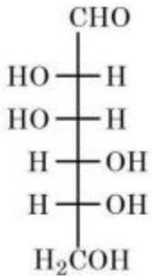
Slaked Lime $\rightarrow Ca(OH)_2$

Dead burnt plaster $\rightarrow CaSO_4$

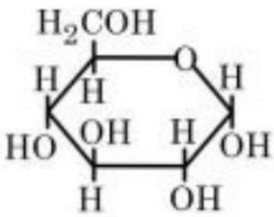
Caustic Soda $\rightarrow NaOH$

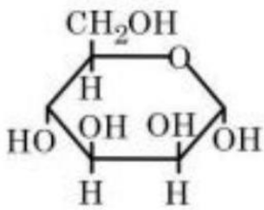
Washing Soda $\rightarrow Na_2CO_3.10H_2O$

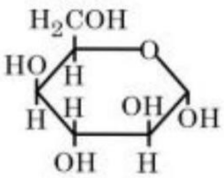
40. The correct representation in six membered pyranose form for the following sugar [X] is

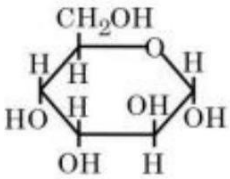


Sugar [X]

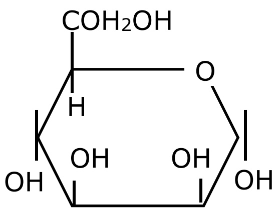
(1) 

(2) 

(3) 

(4) 

Sol. **2**



Haworth structure of mannose

41. Which of the following complex will show largest splitting of d-orbitals ?

(1) $[F_eF_6]^{3-}$ (2) $[F_e(C_2O_4)_3]^{3-}$ (3) $[F_e(CN)_6]^{3-}$ (4) $[F_e(NH_3)_6]^{3+}$

Sol. **3**

(M) Strong field ligands will split ‘d’ orbital largely.

CN^- is SF.L Where as $F^-, C_2O_4^{2-}$ & NH_3

Are comparatively weal feld ligand as common to CN^-

42. Which of the following are the example of double salt?

(A) $FeSO_4 \cdot (NH_4)_2SO_4 \cdot 6H_2O$ (B) $CuSO_4, 4NH_3H_2O$

(C) $K_2SO_4 \cdot Al_2(SO_4)_3 \cdot 24H_2O$ (D) $Fe(CN)_2.4KCN$

Choose the correct answer

(1) B and D only (2) A and C only

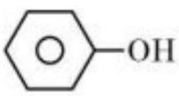
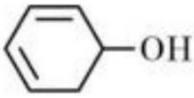
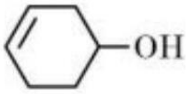
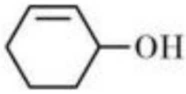
(3) A and B only (4) A, B and D only

Sol. **1**

Double salt contain's two or more types of salts.

$CuSO_4.4NH_3.H_2O$ and $Fe(CN)_2.4KCN$ are complex compounds.

43. Decreasing order of dehydration of the following alcohols is

(a)  (b)  (c)  (d) 

(1) $b > a > d > c$ (2) $a > d > b > c$ (3) $d > b > c > a$ (4) $b > d > c > a$

Sol. **4**

Ease of hydration α stability of carbocation

$b > d > c > a$

44. Given below are two statements:

Statement I: Chlorine can easily combine with oxygen to form oxides; and the product has a tendency to explode.

Statement II: Chemical reactivity of an element can be determined by its reaction with oxygen and halogens.

In the light of the above statements, choose the correct answer from the options given below

(1) Both the Statements I and II are true

(2) Both the Statements I and II are false

(3) Statement I is false but Statement II is true

(4) Statement I is true but Statement II is false

Sol. **1**

Chlorine oxides, Cl_2O, ClO_2, Cl_2O_6 and Cl_2O_7 are heighly Reactive oxidising Agents and tend to explode.

45. Choose the correct statement(s):

A. Beryllium oxide is purely acidic in nature.

B. Beryllium carbonate is kept in the atmosphere of CO_2.

C. Beryllium sulphate is readily soluble in water.

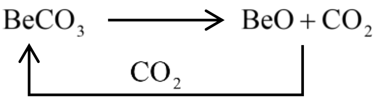
D. Beryllium shows anomalous behavior.

Choose the correct answer from the options given below:

(1) B, C and D only (2) A only (3) A, B and C only (4) A and B only

Sol. **1**

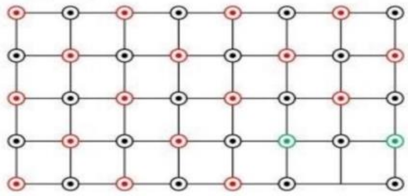
BeO is Amphoteric

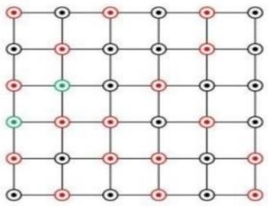


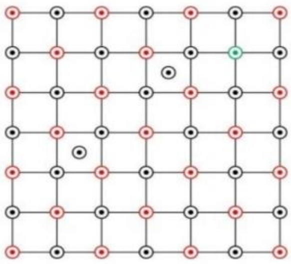
BeSO4 is solube in water

Due to small size Be shows anomalous behaviour.

46. Which of the following represents the lattice structure of $A_{0.95}O$ containing A^{2+}, A^{3+} and O^{2-} ions?
⊙ A^{2+} ⊙ A^{3+} ⊙ O^{2-}

A.


B.


C.


(1) A only (2) B and C only (3) A and B only (4) B only

Sol. **1**

Some vacancy generated by this type defect.

47. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason **R**

Assertion A: In an Ellingham diagram, the oxidation of carbon to carbon monoxide shows a negative slope with respect to temperature.

Reason R: CO tends to get decomposed at higher temperature.

In the light of the above statements, choose the correct answer from the options given below

(1) Both **A** and **R** are correct but **R** is NOT the correct explanation of **A**

(2) Both **A** and **R** are correct and **R** is the correct explanation of **A**

(3) A is correct but **R** is not correct

(4) A is not correct but **R** is correct

Sol. **3**

$2C_{(S)} + O_{2(g)} \longrightarrow 2CO_{(g)}$

$\Delta S°$ is the, $\Delta G° = \Delta H^{e} - T\,\Delta S$

Thus slope is Negative.

As temperature Increase ΔC becomes more Negative thus it has loner tendency to get decomposed.

48. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason **R**

Assertion A: Amongst He, Ne, Ar and Kr; 1 g of activated charcoal adsorbs more of Kr.

Reason R: The critical volume V_c($cm^3\ mol^{-1}$) and critical pressure P_c (atm) is highest for Krypton but the compressibility factor at critical point Z_c is lowest for Krypton.

In the light of the above statements, choose the correct answer from the options given below

(1) **A** is true but **R** is false

(2) Both **A** and **R** are true and **R** is the correct explanation of **A**

(3) A is false but **R** is true

(4) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

Sol. **1**

Assertion A correct but Reason is wrong.

49. Match List I with List II

List I	List II
Test	Functional group / Class of Compound
(A) Molisch's Test	(I) Peptide
(B) Biuret Test	(II) Carbohydrate
(C) Carbylamine Test	(III) Primary amine
(D) Schiff's Test	(IV) Aldehyde

Choose the correct answer from the options given below:

(1) (A) – III, (B) – IV, (C) – I, (D) – II

(2) (A) – II, (B) – I, (C) – III, (D) - IV

(3) (A) – III, (B) – IV, (C) – II, (D) – I

(4) (A) – I, (B) – II, (C) – III, (D) – IV

Sol. **2**

A $\rightarrow$ (II) C $\rightarrow$ (III)

B $\rightarrow$ (I) D $\rightarrow$ (IV)

50. How can photochemical smog be controlled?

(1) By using catalytic convertors in the automobiles/industry.

(2) By complete combustion of fuel.

(3) By using tall chimneys.

(4) By using catalyst.

Sol. **1**

1) By using catalytic convertors in the automobiles / industry.

51. (i) $X(g) \rightleftharpoons Y(g) + Z(g)\ K_{p1} = 3$

(ii) $A(g) \rightleftharpoons 2\,B(g)\ K_{p2} = 1$

If the degree of dissociation and initial concentration of both the reactants X(g) and A(g) are equal, then the ratio of the total pressure at equilibrium $\left(\frac{p_1}{p_2}\right)$ is equal to x : 1. The value of x is___ (Nearest integer)

Sol. $x(g) \rightleftharpoons y(g) + z(g) \qquad Kp_1 = 3$

$$\begin{array}{llll} t=0 & 1 & 0 & 0 \\ teq & 1-x & x & x \\ \text{Partial Pressure} & \frac{(1-x)}{1+x}P_1 & \frac{xP_1}{1+x} & \frac{xP_1}{1+x} \end{array}$$

$A(g) \rightleftharpoons 2B(g)$

$$\begin{array}{lll} t=0 & 1 & 0 \\ teq & 1-x & 2x \\ \text{Partial Pressure} & \frac{1-x}{1+x}\times P_2 & \frac{2x}{1+x}\times P_2 \end{array}$$

$$KP_1 = \frac{\left(\frac{xP_1}{1+x}\right)\left(\frac{xP_1}{1+x}\right)}{\left(\frac{1-x}{1+x}P_1\right)}$$

$$Kp_2 = \frac{(2x)^2 \times P_2^{\,2}}{\left(\frac{1-x}{1+x}\right)P_2}$$

$$\frac{KP_1}{KP_2} = \frac{3}{1} = \frac{P_1}{4P_2}$$

$$\frac{P_1}{P_2} = \frac{12}{1}$$

52. Electrons in a cathode ray tube have been emitted with a velocity of $1000\ \mathrm{m\ s^{-1}}$. The number of following statements which is/are true about the emitted radiation is

Given : $h = 6 \times 10^{-34}\mathrm{Js}, m_e = 9 \times 10^{-31}\ \mathrm{kg}$.

(A) The deBroglie wavelength of the electron emitted is $666.67\ \mathrm{nm}$.

(B) The characteristic of electrons emitted depend upon the material of the electrodes of the cathode ray tube.

(C) The cathode rays start from cathode and move towards anode.

(D) The nature of the emitted electrons depends on the nature of the gas present in cathode ray tube.

Sol. **2**

(A) $\lambda = \frac{h}{mv} = \frac{6\times10^{-34}}{9\times10^{-31}\times1000}$

$= 666.67 \times 10^{-9}\mathrm{m}$

(C) The cathode ray start from Cathode and move towards anode.

53. A and B are two substances undergoing radioactive decay in a container.

The half life of A is 15 min and that of B is 5 min. If the initial concentration of B is 4 times that of A and they both start decaying at the same time, how much time will it take for the concentration of both of them to be same?_____min.

Sol. **15**

Condition $\Rightarrow$ [B] = 4[A]

For A $\quad A \xrightarrow[15\,\mathrm{min}]{t_{1/2}} \frac{A}{2}$

For B $\quad 4A \xrightarrow[15\,\mathrm{min}]{t_{1/2}} 2A \frac{t_{1/2}}{5\,\mathrm{min}} \; A \frac{t_{1/2}}{5\,\mathrm{min}} \; A/2$

54. Sum of oxidation states of bromine in bromic acid and perbromic acid is

Sol. **12**

Bromic Acid $\rightarrow HBrO_5 \rightarrow +5$

Perbromic Acid $\rightarrow HBrO_7 \rightarrow +7$

Sum of oxidation state = 5 + 7 = 12

55. 25 mL of an aqueous solution of KCl was found to require 20 mL of 1M $AgNO_3$ solution when titrated using K_2CrO_4 as an indicator. What is the depression in freezing point of KCl solutions of the given concentration?______(Nearest integer).

(Given: $K_f = 2.0 \text{ K kg mol}^{-1}$)

Assume 1) 100% ionization and

2) density of the aqueous solution as 1 g mL^{-1}

Sol. **3**

$$KCl + AgNO_3 \rightarrow AgCl + KNO_3$$

V = 25ml V = 20ml

M = 1M

At equivalence point,

Mmole of KCl = mmole of $AgNO_3$ = 20 mmole

Volume of solution = 25 ml

Mass of solution = 25 gm

Mass of solvent = 25 – mass of solute

= 25 – [20 $\times$ 10^{-3} $\times$ 74.5]

= 23.51 gm

$$\text{Molality of KCl} = \frac{\text{mole of KCl}}{\text{mass of solvent in kg}}$$

$$= \frac{20 \times 10^{-3}}{23.51 \times 10^{-3}} = 0.85$$

i of KCl = 2 (100% ionisation)

$$\Delta T_f = i \times K_f \times m$$

$$= 2 \times 2 \times 0.85$$

= 3.4

$\simeq 3$

56. At $25^\circ C$, the enthalpy of the following processes are given:

$$H_2(g) + O_2(g) \rightarrow 2OH(g) \quad \Delta H^\circ = 78 \text{ kJ mol}^{-1}$$

$$H_2(g) + 1/2O_2(g) \rightarrow H_2O(g) \quad \Delta H^\circ = -242 \text{ kJ mol}^{-1}$$

$$H_2(g) \rightarrow 2H(g) \quad \Delta H^\circ = 436 \text{ kJ mol}^{-1}$$

$$1/2O_2(g) \rightarrow O(g) \quad \Delta H^\circ = 249 \text{ kJ mol}^{-1}$$

What would be the value of X for the following reaction?

(Nearest integer)

$H_2O(g) \rightarrow H(g) + OH(g) \Delta H^\circ = X \text{kJmol}^{-1}$

Sol. **499**

$$2H_2O(g) \rightarrow H_2(g) + 2(g) \qquad +(242 \times 2)$$

$$H_2(g) + O_2(g) \rightarrow 2OH \qquad +78$$

$$H_2(g) \rightarrow H_2 \qquad +436$$

$$2H_2O \rightarrow 2H + 2OH \qquad +998\text{KJ / mole}$$

$$H_2O \rightarrow H + OH \qquad 998 \times \frac{1}{2} = +499\text{KJ / mole}$$

57. At what pH, given half cell $MnO_4^-(0.1M) \mid Mn^{2+}(0.001M)$ will have electrode potential of 1.282 V ?
(Nearest Integer)
Given $E^o_{MnO_4^-|Mn^{+2}} = 1.54$ V, $\frac{2.303RT}{F} = 0.059$ V

Sol. **3**

$$MnO_4^- + 84^{\oplus} + 5e^{\odot} \rightleftharpoons Mn^{+2} + 4H_2O$$

$$E = E^{\circ} - \frac{0.059}{5}\log\frac{\left[mn^{+2}\right]}{\left[mnO_4^-\right]\left[H^+\right]^8}$$

$$1.282 = 1.54 - \frac{0.059}{5}\log\frac{10^{-3}}{10^{-1}\times\left[H^+\right]}$$

$$\frac{0.258\times5}{0.059} = \log\frac{10^{-2}}{\left[H^+\right]^8}$$

$$21.86 = -2 + 8pH$$

$$pH = 2.98 = 3$$

58. The density of 3M solution of NaCl is 1.0 g mL^{-1}. Molality of the solution is $\times 10^{-2}$ m.
(Nearest integer).
Given: Molar mass of Na and Cl is 23 and 35.5 g mol^{-1} respectively.

Sol. **364**

$$m = \frac{1000\times M}{1000d - M\times M.wt} = \frac{1000\times3}{1000\times1-(3\times58.5)} = 3.64$$

$= 364 \times 10^{-2}$

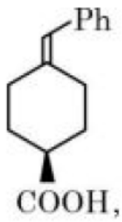
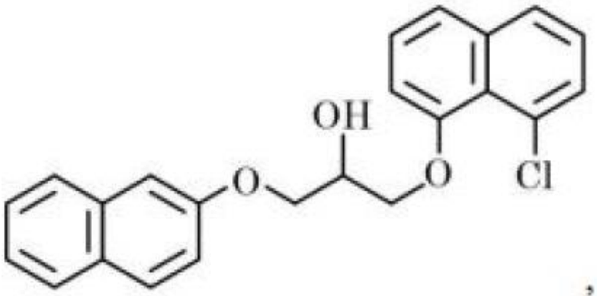
59. Number of isomeric compounds with molecular formula
$C_9H_{10}O$ which (i)do not dissolve in NaOH (ii)do not dissolve in HCl. (iii)do not give orange precipitate with 2,4DNP (iv)on hydrogenation give identical compound with molecular formula $C_9H_{12}O$ is

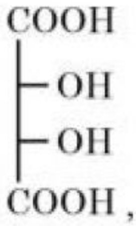
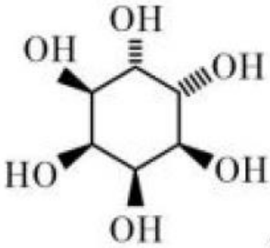
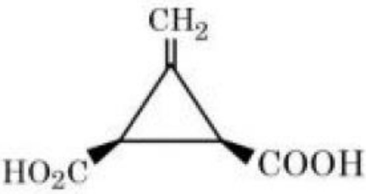
Sol. **2**

$$\underset{D.O.U.=5}{C_9H_{10}O} \longrightarrow \underset{D.O.U.=4}{C_9H_{12}O}$$

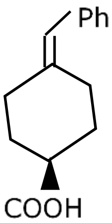
Do not dissolve in NaOH, So no acidic group
Do not dissolve in HCl, So no basic group, no alkene
Do not give orange PPT with 2, 4-DNP so no carbonyl group
Possible compounds – cis and trans of Ph – CH = CH – O – CH_3
(Also Many possible products are there)

60. The total number of chiral compound/s from the following is

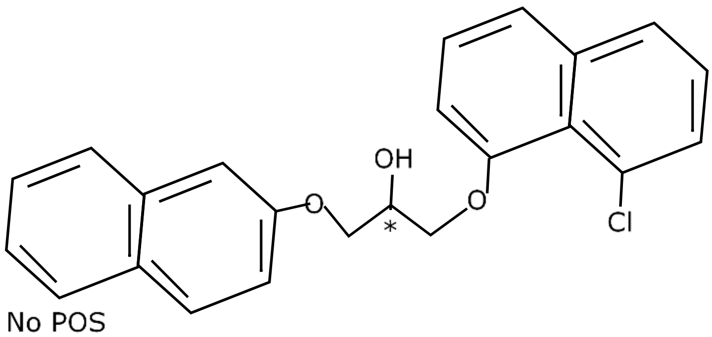
, ,

, , 

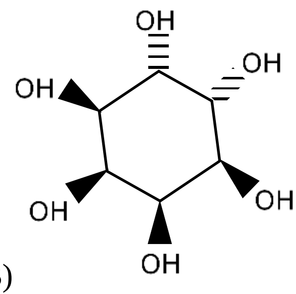
Sol.



(1) No POS
No COS



(2) No POS
No COS



No POS
No COS

(3)

Note :- Take note from gammaxene structure

Mathematics

Section A

61. If $y = y(x)$ is the solution curve of the differential equation
$\frac{dy}{dx} + y\tan x = x\sec x, 0 \le x \le \frac{\pi}{3}, y(0) = 1$, then $y\left(\frac{\pi}{6}\right)$ is equal to

(1) $\frac{\pi}{12} - \frac{\sqrt{3}}{2}\log_e\left(\frac{2\sqrt{3}}{e}\right)$ (2) $\frac{\pi}{12} - \frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$

(3) $\frac{\pi}{12} + \frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$ (4) $\frac{\pi}{12} + \frac{\sqrt{3}}{2}\log_e\left(\frac{2\sqrt{3}}{e}\right)$

Sol. 2

Given D.E. is linear D.E.

I.F. = $e^{\int \tan x dx}$

= $e^{\ell n \sec x}$ = secx

Solution is –

y secx = $\int x\sec^2 x\,dx$

= x tanx – $\int \tan x\,dx$

⇒ y secx = xtanx – ℓn secx + c

Put y(0) = 1

1 = 0 – 0 + c ⇒ c = 1

$Y(x) = \frac{x\tan x}{\sec x} - \frac{\ell n \sec x}{\sec x} + \frac{1}{\sec x}$

$$y\left(\frac{\pi}{6}\right) = \frac{\left(\frac{\pi}{6}\right)\left(\frac{1}{\sqrt{3}}\right)}{\left(\frac{2}{\sqrt{3}}\right)} - \frac{\ell n\left(\frac{2}{\sqrt{3}}\right)}{\left(\frac{2}{\sqrt{3}}\right)} + \frac{\sqrt{3}}{2}$$

$$= \frac{\pi}{12} - \frac{\sqrt{3}}{2}\ell n\left(\frac{2}{\sqrt{3}}\right) + \frac{\sqrt{3}}{2}\ell ne$$

$$= \frac{\pi}{12} - \frac{\sqrt{3}}{2}\ell n\left(\frac{2}{e\sqrt{3}}\right)$$

62. Let R be a relation on $\mathbb{R}$, given by

$$R = \{(a, b): 3a - 3b + \sqrt{7} \text{ is an irrational number }\}.$$

Then R is

(1) an equivalence relation

(2) reflexive and symmetric but not transitive

(3) reflexive but neither symmetric nor transitive

(4) reflexive and transitive but not symmetric

Sol. 3

(a, a) ∈ R ⇒ $3a - 3a + \sqrt{7}$

$= \sqrt{7}$ (irrational)

⇒ R is reflexive

Let $a = \frac{2\sqrt{7}}{3}$ and $b = \frac{\sqrt{7}}{3}$

(a, b) ∈ R ⇒ $2\sqrt{7} - \sqrt{7} + \sqrt{7}$

$= 2\sqrt{7}$ (irration)

(b, a) $\in$ R $\Rightarrow \sqrt{7} - 2\sqrt{7} + \sqrt{7}$

$= 0$ (rational)

$\Rightarrow$ R is no symmetric

Let a = $\frac{2\sqrt{7}}{3}$, b = $\frac{\sqrt{7}}{3}$, C = $\frac{3\sqrt{7}}{3}$

(a ; b) $\in$ R $\Rightarrow 2\sqrt{7}$ (irrational)

(b ; c) $\in$ R $\Rightarrow \sqrt{7}$ (irrational)

(a, c) $\in$ R $\Rightarrow 2\sqrt{7} - 3\sqrt{7} + \sqrt{7}$

$= 0$ (rational)

R is not transitive

$\Rightarrow$ R is reflexive but neither symmetric nor transitive

63. For a triangle ABC, the value of $\cos 2A + \cos 2B + \cos 2C$ is least. If its inradius is 3 and incentre is M, then which of the following is NOT correct?

(1) perimeter of $\triangle ABC$ is $18\sqrt{3}$

(2) $\sin 2A + \sin 2B + \sin 2C = \sin A + \sin B + \sin C$

(3) $\overrightarrow{MA} \cdot \overrightarrow{MB} = -18$

(4) area of $\triangle ABC$ is $\frac{27\sqrt{3}}{2}$

Sol. **4**

Let P $= \cos 2A + \cos 2B + \cos 2C$

$= 2\cos(A + B)\cos(A - B) + 2\cos^2 C - 1$

$= 2\cos(\pi - C)\cos(A - B) + 2\cos^2 C - 1$

$= -2\cos C\,[\cos(A - B) + \cos(A + B)] - 1$

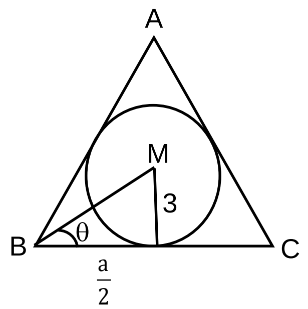
$= -1 - 4\cos A \cos B \cos C$

for P to be minimum

cosA cosB cos C must be maximum

$\Rightarrow \Delta ABC$ is equilateral triangle.

Let side length of triangle is a



$$\tan\theta = \frac{3}{a/2}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{6}{a} \quad \Rightarrow \quad a = 6\sqrt{3}$$

area of triangle = $\frac{\sqrt{3}}{4}a^2$

$$= \frac{\sqrt{3}}{4}(108) = 27\sqrt{3}$$

64. Let S be the set of all solutions of the equation $\cos^{-1}(2x) - 2\cos^{-1}\left(\sqrt{1-x^2}\right) = \pi$, $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$. Then $\sum_{x\in S} 2\sin^{-1}\left(x^2-1\right)$ is equal to

(1) $\pi - 2\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (2) $\pi - \sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

(3) $\frac{-2\pi}{3}$ (4) 0

Sol. **Bonus**

$\cos^{-1}(2x) = \pi + 2\cos^{-1}\sqrt{1-x^2}$

Since $\cos^{-1}(2x) \in [0,\pi]$

R.H.S. $\geq \pi$

$\pi + 2\cos^{-1}\sqrt{1-x^2} = \pi$

$\Rightarrow \cos^{-1}\sqrt{1-x^2} = 0$

$\Rightarrow \sqrt{1-x^2} = 1$

$\Rightarrow x = 0$

but at $x = 0$

$\cos^{-1}(2x) = \cos^{-1}(0) = \frac{\pi}{2}$

no solution possible for given equation.

$x \in \phi$

65. Let S denote the set of all real values of λ such that the system of equations

$\lambda x + y + z = 1$

$x + \lambda y + z = 1$

$x + y + \lambda z = 1$

is inconsistent, then $\sum_{\lambda\in S}\left(|\lambda|^2 + |\lambda|\right)$ is equal to

(1) 4 (2) 12 (3) 6 (4) 2

Sol. **3**

Given system of equation is inconsistent

$\Rightarrow \Delta = 0$

$$\begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$\Rightarrow \lambda^3 - 3\lambda + 2 = 0$

$\Rightarrow (\lambda-1)^2 (\lambda+2) = 0$

$\Rightarrow \lambda = 1, -2$

But for $\lambda = 1$ all planes are same

Then $\lambda = -2$

$\sum_{\lambda\in s}\left(|\lambda|^2 + |\lambda|\right) = 4 + 2 = 6$

66. In a binomial distribution B(n,p), the sum and the product of the mean and the variance are 5 and 6 respectively, then 6(n+p–q) is equal to

(1) 52 (2) 50 (3) 51 (4) 53

Sol. **1**

Given

np + npq = 5

$\Rightarrow$ np(1 + q) = 5 ….(i)

and (np) (npq) = 6

$\Rightarrow$ $n^2p^2q = 6$ ….(ii)

$(i)^2 \div (ii)$

$$\frac{(1+9)^2}{9} = \frac{25}{6}$$

$\Rightarrow$ $6q^2 - 13q + 6 = 0$

$\Rightarrow$ q = $\frac{2}{3}$, $\frac{3}{2}$ (reject)

$P = 1 - \frac{2}{3} = \frac{1}{3}$

$$\frac{n}{3}\left(1+\frac{2}{3}\right) = 5$$

$\Rightarrow$ n = 9

6(n + p – q) = 52

67. The combined equation of the two lines $ax + by + c = 0$ and $a'x + b'y + c' = 0$ can be written as $(ax + by + c)(a'x + b'y + c') = 0$.

The equation of the angle bisectors of the lines represented by the equation

$2x^2 + xy - 3y^2 = 0$ is

(1) $x^2 - y^2 - 10xy = 0$ (2) $x^2 - y^2 + 10xy = 0$

(3) $3x^2 + 5xy + 2y^2 = 0$ (4) $3x^2 + xy - 2y^2 = 0$

Sol. **1**

For pair of st. liens in form

$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$

equation of angle bisector is

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

for $2x^2 + xy - 3y^2 = 0$

a = 2, b = – 3, h = $\frac{1}{2}$

equation of angle bisector is

$$\frac{x^2 - y^2}{5} = \frac{xy}{1/2}$$

$\Rightarrow$ $x^2 - y^2 - 10xy = 0$

68. The area enclosed by the closed curve C given by the differential equation $\frac{dy}{dx}+\frac{x+\alpha}{y-2}=0, y(1)=0$ is 4π.

Let P and Q be the points of intersection of the curve C and the y-axis. If normals at P and Q on the curve C intersect x-axis at points R and S respectively, then the length of the line segment RS is

(1) 2 (2) $\frac{4\sqrt{3}}{3}$ (3) $2\sqrt{3}$ (4) $\frac{2\sqrt{3}}{3}$

Sol. **2**

$$\frac{dy}{dx}+\frac{x+\alpha}{y-2}=0, y(1)=0$$

$$\frac{dy}{dx}=\frac{-(x+\alpha)}{y-2}$$

$$\int (y-2)dy=-\int (x+\alpha)dx$$

$$\frac{y^2}{2}-2y=-\left[\frac{x^2}{2}+\alpha x\right]+\lambda$$

y(1) = 0

$x = 1 \Rightarrow y = 0$

$$0-0=-\left[\frac{1}{2}+\alpha\right]+\lambda$$

$$\frac{y^2}{2}-2y=-\left[\frac{x^2}{2}+\alpha x\right]+\frac{1}{2}+\alpha$$

$$\frac{x^2+y^2}{2}=2y-\alpha x+\frac{1}{2}+\alpha$$

$x^2 + y^2 + 2\alpha x - 4y - 1 - 2\alpha = 0$

Area = 4π

$\pi r^2 = 4\pi$

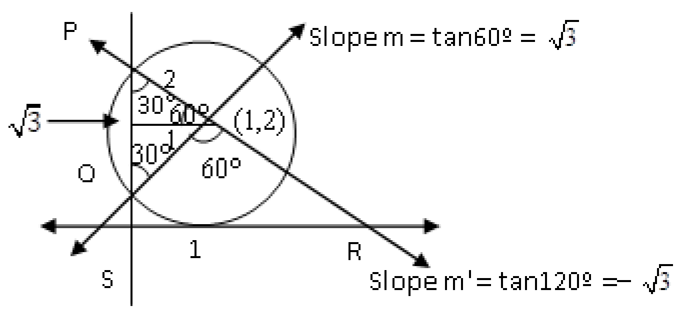
$r^2 = 4$

$\alpha^2 + 4 + 1 + 2\alpha = 4$

$\alpha^2 + 2\alpha + 1 = 0$

$(\alpha + 1)^2 = 0 \Rightarrow [\alpha = -1]$

$x^2 + y^2 - 2x - 4y + 1 = 0$



$y - 2 = \sqrt{3}(x-1)$ $y - 2 = -\sqrt{3}(x-1)$

$y = 0$ $y = 0$

$\frac{-2}{\sqrt{3}} = x - 1$ $\qquad$ $1 + \frac{2}{\sqrt{3}} = x$

$1 - \frac{2}{\sqrt{3}} = x$ $\qquad$ $R\left(1 + \frac{2}{\sqrt{3}}, 0\right)$

$S\left(1 - \frac{2}{\sqrt{3}}, 0\right)$

RS = $\left(1 + \frac{2}{\sqrt{3}}\right) - \left(1 - \frac{2}{\sqrt{3}}\right) = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$

69. The value of

$\frac{1}{1!50!} + \frac{1}{3!48!} + \frac{1}{5!46!} + \cdots . + \frac{1}{49!2!} + \frac{1}{51!1!}$ is :

(1) $\frac{2^{50}}{51!}$ (2) $\frac{2^{51}}{50!}$ (3) $\frac{2^{50}}{50!}$ (4) $\frac{2^{51}}{51!}$

Sol. **1**

S = $\frac{1}{1!50!} + \frac{1}{3!48!} + \frac{1}{5!46!} + \ldots\ldots + \frac{1}{49!2!} + \frac{1}{51!1!}$

$= \frac{1}{51!1!}\left(\frac{51!}{1!50!} + \frac{51!}{3!48!} + \frac{51!}{5!46!} + \ldots\ldots + \frac{51!}{49!2!} + \frac{51!}{51!0!}\right)$

$= \frac{1}{51!1!}\left({}^{51}C_{50} + {}^{51}C_{48} + {}^{51}C_{46} + \ldots\ldots. + {}^{51}C_2 + {}^{51}C_0\right)$

$\because {}^{n}C_0 + {}^{n}C_2 + {}^{n}C_4 + \ldots\ldots. = 2^{n-1}$

S = $\frac{2^{50}}{51!}$

70. The mean and variance of 5 observations are 5 and 8 respectively. If 3 observations are 1, 3, 5 then the sum of cubes of the remaining two observations is

(1) 1216 (2) 1072 (3) 1456 (4) 1792

Sol. **2**

Let remaining two observations are a and b

$5 = \frac{1+3+5+a+b}{5}$

$\Rightarrow$ a + b = 16 …(i)

$8 = \frac{1^2+3^2+5^2+a^2+b^2}{5} - 25$

$\Rightarrow$ $a^2 + b^2 = 130$

$a^2 + b^2 = 130$ …(ii)

$(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow$ 256 = 130 + 2ab

ab = 63

$a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

$= (16)^3 - 3(63)(16)$

$= 4096 - 3024$

$\Rightarrow a^3 + b^3 = 1072$

71. The sum to 10 terms of the series

$\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+\cdots$. is

(1) $\frac{55}{111}$ (2) $\frac{56}{111}$ (3) $\frac{58}{111}$ (4) $\frac{59}{111}$

Sol. 1

$$T_n=\frac{n}{1+n^2+n^4}$$

$$=\frac{n}{(n^2-n+1)(n^2+n+1)}$$

$$=\frac{1}{2}\left[\frac{(n^2+n+1)-(n^2-n+1)}{(n^2-n+1)(n^2+n+1)}\right]$$

$$\Rightarrow T_n=\frac{1}{2}\left[\frac{1}{(n^2-n+1)}-\frac{1}{(n^2+n+1)}\right]$$

$$S_n=\sum_{n=1}^{10}T_n$$

$$=\frac{1}{2}\sum\left(\frac{1}{n^2-n+1}-\frac{1}{n^2+n+1}\right)$$

$$=\frac{1}{2}\left[\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{13}\right)\right.$$

$$\left.\ldots\ldots\ldots\ldots\ldots\ldots+\left(\frac{1}{91}-\frac{1}{111}\right)\right]$$

$$=\frac{1}{2}\left[1-\frac{1}{111}\right]=\frac{55}{111}$$

72. The shortest distance between the lines

$\frac{x-5}{1}=\frac{y-2}{2}=\frac{z-4}{-3}$ and $\frac{x+3}{1}=\frac{y+5}{4}=\frac{z-1}{-5}$ is

(1) $5\sqrt{3}$ (2) $7\sqrt{3}$ (3) $6\sqrt{3}$ (4) $4\sqrt{3}$

Sol. 3

$$L_1:\frac{x-5}{1}=\frac{y-2}{2}=\frac{z-4}{-3}$$

$$\vec{a_1}=5\hat{i}+2\hat{j}+4\hat{k}$$

$$\vec{r_1}=\hat{i}+2\hat{j}-3\hat{k}$$

$$L_2:\frac{x+3}{1}=\frac{y+5}{4}=\frac{z-1}{-5}$$

$$\vec{a_2}=-3\hat{i}-5\hat{j}+\hat{k}$$

$$\vec{r_2}=\hat{i}+4\hat{j}-5\hat{k}$$

$$\vec{r_1}\times\vec{r_2}=\begin{vmatrix}\hat{i} & j & k\\ 1 & 2 & -3\\ 1 & 4 & -5\end{vmatrix}$$

$$=2\hat{i}+2\hat{j}+2\hat{k}$$

Shortest distance (d) = $\left|\dfrac{(\vec{r_1}\times\vec{r_2}).(\vec{a_1}-\vec{a_2})}{|\vec{r_1}\times\vec{r_2}|}\right|$

$= \dfrac{36}{2\sqrt{3}} = 6\sqrt{3}$

73. $\lim_{n\to\infty}\left[\frac{1}{1+n}+\frac{1}{2+n}+\frac{1}{3+n}+\cdots+\frac{1}{2n}\right]$ is equal to

(1) $\log_e 2$ (2) $\log_e\left(\frac{3}{2}\right)$ (3) $\log_e\left(\frac{2}{3}\right)$ (4) 0

Sol. **1**

$\lim_{n\to\infty}\left[\dfrac{1}{1+n}+\dfrac{1}{2+n}+\dfrac{1}{3+n}+......+\dfrac{1}{2n}\right]$

$= \lim_{n\to\infty}\sum_{r=1}^{n}\dfrac{1}{r+n}$

$= \lim_{n\to\infty}\sum_{r=1}^{n}\dfrac{1}{n}\left(\dfrac{1}{\frac{r}{n}+1}\right)$

$= \int_0^1 \dfrac{dx}{x+1}$

$= \log_e(1+\text{x})\Big|_0^1$

$= \log_e^2$

74. Let the image of the point $P(2,-1,3)$ in the plane $x+2y-z=0$ be Q. Then the distance of the plane $3x+2y+z+29=0$ from the point Q is

(1) $\frac{24\sqrt{2}}{7}$ (2) $2\sqrt{14}$ (3) $3\sqrt{14}$ (4) $\frac{22\sqrt{2}}{7}$

Sol. **3**

let Q(α, β, γ) is image of P(2, –1, 3) in the plane x + 2y – z = 0

$\dfrac{\alpha-2}{1} = \dfrac{\beta+1}{2} = \dfrac{\gamma-3}{-1} = \dfrac{-2(2-2-3)}{1^2+2^2+(-1)^2} = 1$

α = 3, β = 1, γ = 2

Distance of Q(3, 1, 2) from

3x + 2y + z + 29 = 0

D = $\dfrac{|3(3)+2(1)+2+29|}{\sqrt{3^2+2^2+1^2}}$

$= \dfrac{42}{\sqrt{14}} = 3\sqrt{14}$

75. Let $f(x) = 2x + \tan^{-1} x$ and $g(x) = \log_e\left(\sqrt{1+x^2} + x\right), x \in [0,3]$.
Then
(1) $\min f'(x) = 1 + \max g'(x)$
(2) $\max f(x) > \max g(x)$
(3) there exist $0 < x_1 < x_2 < 3$ such that $f(x) < g(x), \forall x \in (x_1, x_2)$
(4) there exists $\hat{x} \in [0,3]$ such that $f'(\hat{x}) < g'(\hat{x})$

Sol. **2**

$f'(x) = 2 + \frac{1}{1+x^2} > 0$ for $x \in [0, 3]$

$f(x)\uparrow$ for $x \in [0, 3]$

$f(0) = 0$, $f(3) = 6 + \tan^{-1}(3)$

$$g'(x) = \frac{\frac{x}{\sqrt{x^2+1}} + 1}{x + \sqrt{1+x^2}} = \frac{1}{\sqrt{1+x^2}} > 0 \quad \text{for } x \in [0, 3]$$

$g(x)\uparrow$ for $x \in [0, 3]$

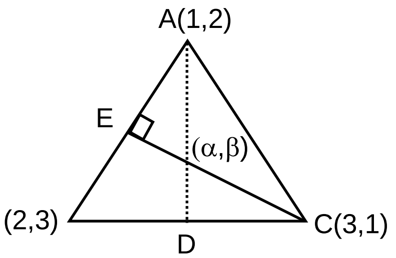
$g(0) = 0$, $g(3) = log_e\left(\sqrt{10} + 3\right)$

$\max f(x) > \max g(x)$

Option (2) correct

76. If the orthocentre of the triangle, whose vertices are (1,2) (2,3) and (3,1) is (α,β), then the quadratic equation whose roots are $\alpha + 4\beta$ and $4\alpha + \beta$, is
(1) $x^2 - 20x + 99 = 0$ (2) $x^2 - 19x + 90 = 0$
(3) $x^2 - 22x + 120 = 0$ (4) $x^2 - 18x + 80 = 0$

Sol. **1**



equation of AD : $x - 2y + 3 = 0$

equation of CE : $x + y - 4 = 0$

orthocenter (α, β) is $\left(\frac{5}{3}, \frac{7}{3}\right)$

$\alpha + 4\beta = 11$ and $4\alpha + \beta = 9$

Quadratic equation is

$x^2 - (11 + 9)x + (11 \times 9) = 0$

$\Rightarrow x^2 - 20x + 99 = 0$

77. Let $S = \{x : x \in \mathbb{R} \text{ and } (\sqrt{3}+\sqrt{2})^{x^2-4} + (\sqrt{3}-\sqrt{2})^{x^2-4} = 10\}$
Then $n(S)$ is equal to
(1) 4 (2) 0 (3) 6 (4) 2

Sol. **1**

$$\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4} + \left(\sqrt{3}-\sqrt{2}\right)^{x^2-4} = 10$$

$$\Rightarrow \left(\sqrt{3}+\sqrt{2}\right)^{x^2-4} + \frac{1}{\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}} = 10$$

Let $\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4} = t$

$t + \dfrac{1}{t} = 10$

$\Rightarrow t^2 - 10t + 1 = 0$

$t = 5+2\sqrt{6},\ 5-2\sqrt{6}$

If $t = 5+2\sqrt{6}$

$\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4} = \left(\sqrt{3}+\sqrt{2}\right)^2$

$\Rightarrow x^2 - 4 = 2$

$\Rightarrow x = \pm\sqrt{6}$

If $t = 5-2\sqrt{6}$

$\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4} = \left(\sqrt{3}+\sqrt{2}\right)^{-2}$

$\Rightarrow x^2 - 4 = -2$

$\Rightarrow x = \pm\sqrt{2}$

S = $\left\{\sqrt{6}, -\sqrt{6}, \sqrt{2}, -\sqrt{2}\right\}$

n (s) = 4

78. If the center and radius of the circle $\left|\dfrac{z-2}{z-3}\right| = 2$ are respectively (α, β) and γ.
then $3(\alpha + \beta + \gamma)$ is equal to
(1) 11 (2) 12 (3) 9 (4) 10

Sol. **2**

Put z = x + iy

$$\frac{|(x-2)+iy|}{|(x-3)+iy|} = 2$$

$\Rightarrow (x-2)^2 + y^2 = 4((x-3)^2 + y^2)$

$\Rightarrow x^2 - 4x + 4 + y^2 = 4x^2 - 24x + 36 + 4y^2$

$\Rightarrow 3x^2 + 3y^2 - 20x + 32 = 0$

$\Rightarrow x^2 + y^2 - \dfrac{20}{3}x + \dfrac{32}{3} = 0$

Center $(\alpha, \beta) = \left(\dfrac{10}{3}, 0\right)$

Radius $(\gamma) = \sqrt{\left(-\dfrac{10}{3}\right)^2 - \dfrac{32}{3}} = \dfrac{2}{3}$

$$3\left(\frac{10}{3} + 0 + \frac{2}{3}\right) = 12$$

79. Let $f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1+\cos^2 x & \sin 2x \\ \sin^2 x & \cos^2 x & 1+\sin 2x \end{vmatrix}$, $x \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$. If α and β respectively are the maximum and the minimum values of f, then

(1) $\alpha^2 + \beta^2 = \frac{9}{2}$ (2) $\beta^2 - 2\sqrt{\alpha} = \frac{19}{4}$ (3) $\alpha^2 - \beta^2 = 4\sqrt{3}$ (4) $\beta^2 + 2\sqrt{\alpha} = \frac{19}{4}$

Sol. **2**

$$f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1+\cos^2 x & \sin 2x \\ \sin^2 x & \cos^2 x & 1+\sin x \end{vmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$= \begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

$= (1 + \sin^2 x) - \cos^2 x(-1) + \sin 2x$

$f(x) = 2 + \sin 2x$

$$2x \in \left[\frac{\pi}{3}, \frac{2\pi}{3}\right] \Rightarrow \frac{\sqrt{3}}{2} \le \sin 2x \le 1$$

$\alpha = 2 + 1 = 3$

$$\beta = 2 + \frac{\sqrt{3}}{2}$$

$$\beta^2 - 2\sqrt{\alpha} = \left(2 + \frac{\sqrt{3}}{2}\right)^2 - 2\sqrt{3}$$

$$= 4 + \frac{3}{4} + 2\sqrt{3} - 2\sqrt{3}$$

$$= \frac{19}{4}$$

80. The negation of the expression $q \vee ((\sim q) \wedge p)$ is equivalent to

(1) $(\sim p) \vee (\sim q)$ (2) $p \wedge (\sim q)$ (3) $(\sim p) \vee q$ (4) $(\sim p) \wedge (\sim q)$

Sol. **4**

$\sim (q \vee (\sim q) \wedge p)$

$= \sim q \wedge (q \vee \sim p)$

$= (\sim q \wedge q) \vee (\sim q \wedge \sim p)$

$= F \vee (\sim q \wedge \sim p) = (\sim q) \wedge (\sim p)$

Section B

81. Let $\vec{v} = \alpha\hat{i} + 2\hat{j} - 3\hat{k}, \vec{w} = 2\alpha\hat{i} + \hat{j} - \hat{k}$ and $\vec{u}$ be a vector such that $|\vec{u}| = \alpha > 0$. If the minimum value of the scalar triple product $[\vec{u}\vec{v}\vec{w}]$ is $-\alpha\sqrt{3401}$, and $|\vec{u} \cdot \hat{i}|^2 = \frac{m}{n}$ where m and n are coprime natural numbers, then $m + n$ is equal to

Sol. **3501**

$$\vec{v}\times\vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 2 & -3 \\ 2\alpha & 1 & -1 \end{vmatrix}$$

$= \hat{i} - 5\alpha\hat{j} - 3\alpha\hat{k}$

$[u\ v\ w] = \vec{u}.(\vec{v}\times\vec{w})$

$= |\vec{u}|\,|\vec{v}\times\vec{w}|\cos\theta$

since [u v w] is Least $\Rightarrow \cos\theta = -1$

$[u\ v\ w] = \left(|\vec{u}|\sqrt{1+25\alpha^2+9\alpha^2}\right)(-1)$

$\Rightarrow -\alpha\sqrt{1+34\alpha^2} = -\alpha\sqrt{3401}$

$\Rightarrow \alpha^2 = 100$

$\Rightarrow \alpha = 10 \qquad \{\because \alpha > 0\}$

$\vec{u}$ is parallel to $\vec{v}\times\vec{w}$

$\vec{u} = \lambda(\vec{v}\times\vec{w})$

$\vec{u} = \lambda(\hat{i} - 50\hat{j} - 30\hat{k})$

$|\vec{u}| = 10$

$|\lambda|\sqrt{3401} = 10$

$|\lambda| = \dfrac{10}{\sqrt{3401}} \qquad \vec{u} = \pm\dfrac{10}{\sqrt{3401}}\left(\hat{i} - 50\hat{j} - 30\hat{k}\right)$

$|\vec{u}.\hat{i}|^2 = \dfrac{100}{3401} = \dfrac{m}{n}$

$m + n = 100 + 3401 = 3501$

82. The number of words, with or without meaning, that can be formed using all the letters of the word ASSASSINATION so that the vowels occur together, is

Sol. **50400**

A – 3, I – 2, S – 4, N – 2, O –1, T–1

As vowels are together

Total words formed = $\left(\dfrac{8!}{4!2!}\right)\left(\dfrac{6!}{3!2!}\right)$

$= \left(\dfrac{8\times7\times6\times5}{2}\right)\left(\dfrac{6\times5\times4}{2}\right) = 50400$

83. The remainder, when $19^{200} + 23^{200}$ is divided by 49 , is

Sol. **29**

$19^{200} + 23^{200} \quad a^{n} + b^{n}$

$19^3 = 6859 = 140 \times 49 - 1$

$= 49\lambda - 1$

$(19^3)^{66} = (49\lambda - 1)^{66}$

So, Remainder of 19^{198} divided by 49

is $(-1)^{66} = 1$

$19^2 = 361$ gives remainder 18
So, 19^{200} gives remainder 18
23^2 gives remainder 39
$(23)^3$ gives remainder 15
$(23)^4$ gives remainder 2
$((23)^4)^6$ gives remainder $(2)^6 = 64$
& 64 gives remainder 15
$(23)^{24} \longrightarrow 15$
$(23)^{25} \longrightarrow 2$
$((23)^{25})^8 \longrightarrow (2)^8 = 256 \longrightarrow 11$
So, Total remainder = 18 + 11 = 29

84. The number of 3-digit numbers, that are divisible by either 2 or 3 but not divisible by 7 , is
Sol. **514**
3 digit numbers divisible by either 2 or 3
P = n(divisible by 2) + n(divisible by 3) – n(divisible by 6)
P = 450 + 300 – 150
P = 600
Q = n(divisible by 14) + n(divisible by 21) – n(divisible by 42)
= 64 + 43 – 21 = 86
3 digit number divisible by either 2 or 3
But not divisible by –1 so P – Q = 600 – 86 = 514

85. Let $f:\mathbb{R} \to \mathbb{R}$ be a differentiable function such that $f'(x) + f(x) = \int_0^2 f(t)dt$.
If $f(0) = e^{-2}$, then $2f(0) - f(2)$ is equal to
Sol. **1**
Let $\int_0^2 f(t)dt = \lambda$
$f'(x) + f(x) = \lambda$
is linear Differential equation
I.f. = $e^{\int dx} = e^x$
$f(x).e^x = \int e^x \lambda dx$
$\Rightarrow f(x).e^x = \lambda e^x + C$
$\Rightarrow f(x) = \lambda + Ce^{-x}$
put $f(0) = e^{-2}$
$e^{-2} = \lambda + C \Rightarrow C = e^{-2} - \lambda$
$f(x) = \lambda + (e^{-2} - \lambda)e^{-x}$
$\lambda = \int_0^2 f(t)dt$
$= \int_0^2 (\lambda + (e^{-2} - \lambda)e^{-t})dt$
$\Rightarrow \lambda = \lambda + \lambda e^{-2} - e^{-4} + e^{-2}$
$\Rightarrow \lambda = e^{-2} - 1$
$f(x) = e^{-2} - 1 + e^{-x}$

$f(0) = e^{-2}$
$f(2) = 2e^{-2} - 1$
$2f(0) - f(2) = 1$

86. If $f(x) = x^2 + g'(1)x + g''(2)$ and $g(x) = f(1)x^2 + xf'(x) + f''(x)$, then the value of $f(4) - g(4)$ is equal to

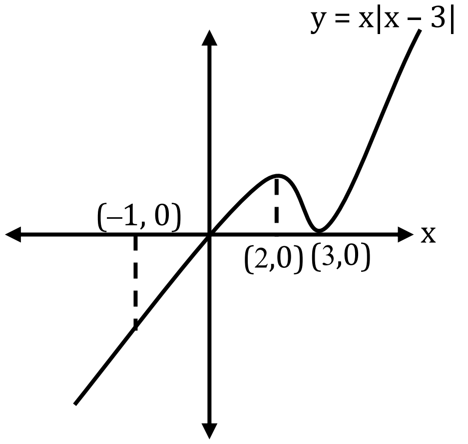
Sol. **14**

let $g'(1) = A$
$g''(2) = B$
$f(x) = x^2 + Ax + B$
$f(1) = A + B + 1$
$f'(x) = 2x + A$
$f''(x) = 2$
$g(x) = (A + B + 1)\,x^2 + x(2x + A) + 2$
$\Rightarrow g(x) = x^2(A + B + 2) + Ax + 2$
$g'(x) = 2x(A + B + 2) + A$
$g'(1) = A$
$\Rightarrow 2(A + B + 2) + A = A$
$A + B = -2$ ....(i)
$g''(x) = 2(A + B + 2)$
$g''(2) = B$
$\Rightarrow 2(A + B + 2) = B$
$\Rightarrow 2A + B = -4$ ...(ii)
From (i) and (ii)
$A = -2$ and $B = 0$
$f(x) = x^2 - 2x$
$f(4) = 16 - 8 = 8$
$g(x) = -2x + 2$
$g(4) = -8 + 2 = -6$
$f(4) - g(4) = 8 - (-6) = 14$

87. Let A be the area bounded by the curve $y = x|x - 3|$, the x-axis and the ordinates $x = -1$ and $x = 2$. Then $12A$ is equal to

Sol. **62**

$$y = x|x-3| = \begin{cases} x(x-3); & x \geq 3 \\ x(3-x); & x < 3 \end{cases}$$



$A = -\int_{-1}^{0} x(3-\mathrm{x})\,\mathrm{dx} + \int_{0}^{2} x(3-\mathrm{x})\,\mathrm{dx}$

$= \int_{-1}^{0} (x^2 - 3\mathrm{x})\,\mathrm{dx} + \int_{0}^{2} (3x - x^2)\,\mathrm{dx}$

$= \left[\frac{x^3}{3} - \frac{3x^2}{2}\right]_{-1}^{0} + \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_{0}^{2}$

$A = 0 - \left(\frac{-1}{3} - \frac{3}{2}\right) + 6 - \frac{8}{3} = \frac{31}{6}$

$A = 12\left(\frac{31}{6}\right) = 62$

88. If $\int_0^1 (x^{21} + x^{14} + x^7)(2x^{14} + 3x^7 + 6)^{1/7} dx = \frac{1}{l}(11)^{m/n}$ where $l, m, n \in \mathbb{N}, m$ and n are coprime then $l + m + n$ is equal to

Sol. **63**

$\mathrm{I} = \int_{0}^{1} (\mathrm{x}^{21} + \mathrm{x}^{14} + \mathrm{x}^{7})\ (2\mathrm{x}^{14} + 3\mathrm{x}^{7} + 6)^{\frac{1}{7}}\,\mathrm{dx}$

$= \int_{0}^{1} (\mathrm{x}^{20} + \mathrm{x}^{13} + \mathrm{x}^{6})\ (2\mathrm{x}^{21} + 3\mathrm{x}^{14} + 6x^{7})^{\frac{1}{7}}\,\mathrm{dx}$

Put $2x^{21} + 3x^{14} + 6x^{7} = t$

$\Rightarrow 42(x^{20} + x^{13} + x^{6})dx = dt$

$\Rightarrow (x^{20} + x^{13} + x^{6})dx = \frac{dt}{42}$

$\mathrm{I} = \int_{0}^{11} \frac{t^{\frac{1}{7}}}{42}\,dt$

$= \frac{1}{42}\left[\frac{t^{\frac{8}{7}}}{\frac{8}{7}}\right]_{0}^{11}$

$= \left(\frac{7}{8}\right)\left(\frac{1}{42}\right)(11)^{8/7}$

$= \frac{1}{48}(11)^{8/7} = \frac{1}{\ell}(11)^{\mathrm{m/n}}$

$\ell + \mathrm{m} + \mathrm{n} = 48 + 8 + 7 = 63$

89. Let $a_1 = 8, a_2, a_3, \ldots, a_n$ be an A.P. If the sum of its first four terms is 50 and the sum of its last four terms is 170 , then the product of its middle two terms is

Sol. **754**

$a_1 = 8$

d = common difference

$\frac{4}{2}$ [16 + 3d] = 50

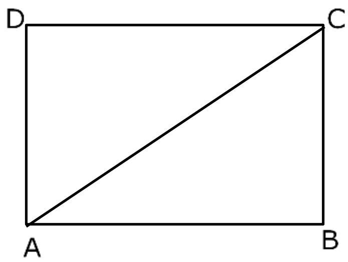
$\Rightarrow$ d = 3

$\frac{4}{2}$ $[2a_n + 3(-d)] = 170$
$\Rightarrow 2(a_1 + (n - 1)d) - 3d = 85$
$\Rightarrow 16 + 6(n - 1) - 9 = 85$
$n - 1 = 13$
$n = 14$
Product of middle two terms = $T_7 \times T_8$
$= (a_1 + 6d)(a_1 + 7d)$
$= (8 + 18)(8 + 21)$
$= (26)(29) = 754$

90. $A(2,6,2), B(-4,0,\lambda), C(2,3,-1)$ and $D(4,5,0), |\lambda| \leq 5$ are the vertices of a quadrilateral $ABCD$. If its area is 18 square units, then $5 - 6\lambda$ is equal to

Sol. **11**

$\overrightarrow{AD} = 2\hat{i} - \hat{j} - 2\hat{k}$
$\overrightarrow{AC} = -3j - 3\hat{k}$



$$\overrightarrow{AD} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & -2 \\ 0 & -3 & -3 \end{vmatrix}$$
$= -3\hat{i} + 6\hat{j} - 6\hat{k}$
Area (ΔADC) = $\frac{1}{2}\left|\overrightarrow{AD} \times \overrightarrow{AC}\right|$
$= \frac{1}{2}\sqrt{9 + 36 + 36} = \frac{9}{2}$
$\overrightarrow{AB} = -6\hat{i} - 6\hat{j} + (\lambda - 2)\hat{k}$
$\overrightarrow{AC} = -3\hat{j} - 3\hat{k}$
$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -6 & -6 & \lambda - 2 \\ 0 & -3 & -3 \end{vmatrix}$$
$= (12 + 3\lambda)\hat{i} - 18\hat{j} + 18\hat{k}$
area (ΔABC) = $\frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$
$= \frac{3}{2}\sqrt{(4 + \lambda)^2 + 36 + 36}$
Area(ΔABCD) = ar(ΔADC) + ar(ΔABC)
$\Rightarrow 18 = \frac{9}{2} + \frac{3}{2}\sqrt{(4 + \lambda)^2 + 72}$
$\Rightarrow (4 + \lambda)^2 = 9$
$4 + \lambda = 3$ or $4 + \lambda = -3$
$\Rightarrow \lambda = -1$ or $\lambda = -7$ (reject)
$5 - 6\lambda = 5 + 6 = 11$