JEE–MAIN EXAMINATION – JANUARY, 2023	
(Held On Thursday 30th January, 2023)	TIME : 9 : 00 AM to 12 : 00 NOON

Physics

SECTION - A

1. The magnetic moments associated with two closely wound circular coils A and B of radius $r_A = 10$ cm and $r_B = 20$ cm respectively are equal if : (Where N_A, I_A and N_B, I_B are number of turn and current of A and B respectively)

(1) $4 N_A I_A = N_B I_B$ (2) $N_A = 2 N_B$ (3) $N_A I_A = 4 N_B I_B$ (4) $2 N_A I_A = N_B I_B$

Sol. **(3)**

Magnetic moment m = IAN

Magnetic moment of coil A →

$$m_A = I_A \pi r_A^2 N_A$$

$$m_A = I_A \pi N_A (10)^2 \quad ...(1)$$

Magnetic moment of coil B →

$$m_B = I_B N_B \pi r_B^2$$

$$m_B = I_B N_B \pi (20)^2 \quad ...(2)$$

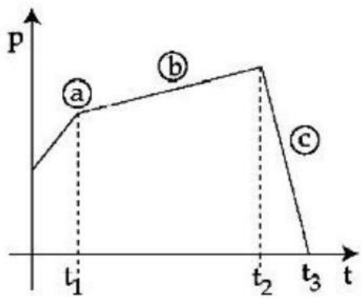
Now $m_A = m_B$

$$I_A . \pi N_A (100) = I_B N_B \pi 400$$

$$\boxed{I_A N_A = 4 I_B N_B}$$

2. The figure represents the momentum time (p-t) curve for a particle moving along an axis under the influence of the force. Identify the regions on the graph where the magnitude of the force is maximum and minimum respectively ?

If $(t_3 - t_2) < t_1$



(1) c and b (2) b and c (3) *a* and *b* (4) c and a

Sol. **(1)**

Slope of curve P-t will represent the force so

$$F = \frac{dP}{dt} = \text{slope}$$

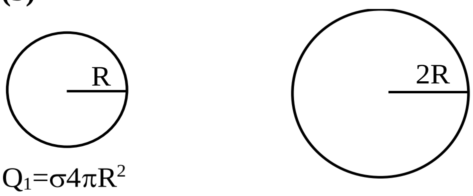
Maximum slope → (c)

Minimum slope → (b)

3. Two isolated metallic solid spheres of radii R and 2R are charged such that both have same charge density σ. The spheres are then connected by a thin conducting wire. If the new charge density of the bigger sphere is σ'. The ratio $\frac{\sigma'}{\sigma}$ is :

(1) $\frac{4}{3}$ (2) $\frac{5}{3}$ (3) $\frac{5}{6}$ (4) $\frac{9}{4}$

Sol. **(3)**

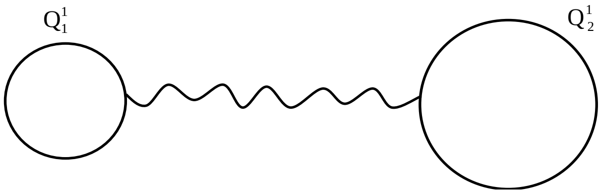


$Q_2 = \sigma 4\pi(2R)^2$

$Q_2 = \sigma 16\pi R^2$

$Q_1 = \sigma 4\pi R^2$

Now



Charge will flow until voltage of both sphere become equal so

$$c = 4\pi\varepsilon_0 R$$

$$v_1^1 = v_2^1$$

$$\frac{Q_1}{c_1} = \frac{Q_2}{c_2} \Rightarrow \frac{Q_1'}{4\pi\varepsilon_0 R} = \frac{Q_2'}{4\pi\varepsilon_0(2R)}$$

$$\Rightarrow \quad 2Q_1' = Q_2' \quad \ldots(1)$$

$$Q_1 + Q_2 = Q_1' + Q_2'$$

$$\sigma 20\pi R^2 = Q_2' + \frac{Q_2'}{2} = \frac{3}{2}Q_2' \Rightarrow Q_2' = \frac{\sigma 40\pi R^2}{3} \quad \ldots(2)$$

$$Q_2' = \frac{\sigma 40\pi R^2}{3}$$

Now $\sigma' 4\pi(2R)^2 = \dfrac{\sigma 40\pi R^2}{3}$

$$\sigma' 16\pi R^2 = \frac{\sigma 40\pi R^2}{3}$$

$$\frac{\sigma'}{\sigma} = \frac{40}{3} \times \frac{1}{16} = \frac{5}{6}$$

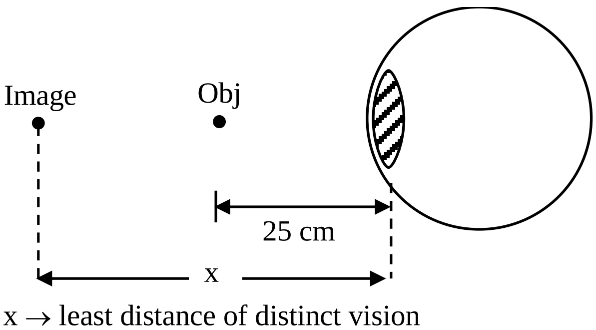
4. A person has been using spectacles of power −1.0 dioptre for distant vision and a separate reading glass of power 2.0 dioptres. What is the least distance of distinct vision for this person :

(1) 40 cm (2) 30 cm (3) 10 cm (4) 50 cm

Sol. **(4)**

Power convex lens = 2.0 D

Power of concave lens = –1.0 D



x → least distance of distinct vision

$f = \frac{1}{2} \times 100 = 50cm$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{50} = \frac{1}{(-x)} - \frac{1}{-25} \quad \Rightarrow \quad \frac{1}{50} - \frac{1}{25} = \frac{1}{(-x)}$$

$$\Rightarrow \quad \frac{1-2}{50} = \frac{-1}{x}$$

$$\Rightarrow \quad \boxed{x = 50cm}$$

5. A small object at rest, absorbs a light pulse of power 20 mW and duration 300 ns. Assuming speed of light as 3×10^8 m/s, the momentum of the object becomes equal to :

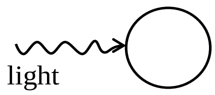
(1) 3×10^{-17} kg m/s (2) 2×10^{-17} kg m/s (3) 1×10^{-17} kg m/s (4) 0.5×10^{-17} kg m/s

Sol. **(2)**

Power = 20 mw

t = 300 nsec

energy absorbed $= 300 \times 10^{-9} \times 20 \times 10^{-3} = 6 \times 10^{3} \times 10^{-12} = 6 \times 10^{-9}$ J



Pressure = $\frac{\text{Intensity}}{C} = \frac{\text{Power}}{\text{Area} \times C}$

Pressure × Area = $\frac{\text{Power}}{C}$

Force = $\frac{\text{Power}}{C} = \frac{20 \times 10^{-3}}{3 \times 10^{8}}$

$F = \frac{20}{3} \times 10^{-11}$ N

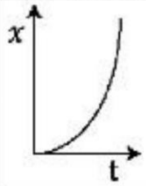
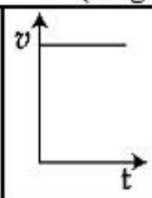
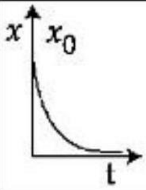
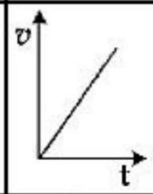
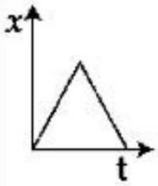
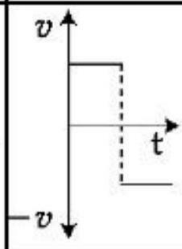
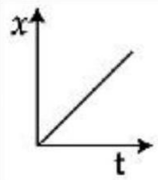
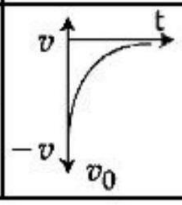
$F\Delta t = \Delta P$ (momentum)

$$\frac{20}{3} \times 10^{-11} \times 300 \times 10^{-9} = P_f - P_i$$

$$20 \times 10^{-20} \times 100 = P_f$$

$$\boxed{2 \times 10^{-17} = P_f}$$

6. Match Column-I with Column-II :

	Column-I (x-t graphs)		Column-II (v-t graphs)
A.		I.	
B.		II.	
C.		III.	
D.		IV.	

Choose the correct answer from the options given below:

(1) A- I, B-II, C-III, D-IV

(2) A- II, B-III, C-IV, D-I

(3) A- I, B-III, C-IV, D-II

(4) A- II, B-IV, C-III, D-I

Sol. **(4)**

(A) $x \propto t^2$

$\frac{dx}{dt} \propto 2t \Rightarrow \boxed{V \propto t}$ $\quad$ $A \rightarrow II$

(B) $x = x_0 e^{-\alpha t}$

$\frac{dx}{dt} = x_0 e^{-\alpha t}(-\alpha) = -\alpha(x_0 e^{-\alpha t})$

$V = -\alpha x_0 e^{-\alpha t}$ $\quad$ $B \rightarrow IV$

(C) $x \propto t \rightarrow V = \text{const}$

$x \propto -t \rightarrow V = -\text{const}$ $\quad$ $C \rightarrow III$

(D) $x \propto t \rightarrow V = \text{const}$ $\quad$ $D \rightarrow I$

7. The pressure (P) and temperature (T) relationship of an ideal gas obeys the equation PT^2 = constant. The volume expansion coefficient of the gas will be :

(1) $\frac{3}{T^3}$ (2) $\frac{3}{T^2}$ (3) $3\,T^2$ (4) $\frac{3}{T}$

Sol. **(4)**

$PT^2 = \text{const.}$

$dV = V\gamma dT$

$\gamma = \frac{1}{V}\frac{dV}{dT}$ ...(1)

Using PV = nRT and PT^2 = cont.

$\frac{nRT}{V}.T^2 = \text{const}$

$V \propto T^3 \Rightarrow V = KT^3$ ...(2)

Now put in (1)

$\gamma = \frac{1}{KT^3} \times 3KT^2 = \frac{3}{T} \Rightarrow \gamma = \frac{3}{T}$

8. Heat is given to an ideal gas in an isothermal process.
A. Internal energy of the gas will decrease.
B. Internal energy of the gas will increase.
C. Internal energy of the gas will not change.
D. The gas will do positive work.
E. The gas will do negative work.
Choose the correct answer from the options given below :
(1) C and D only (2) C and E only (3) A and E only (4) B and D only

Sol. **(1)**

In isothermal process

$\Delta T = 0$

So $\Delta U = 0$

$\Delta Q = \omega + \Delta U$

$\boxed{\Delta Q = \omega}$

So heat will be used to do positive work

9. If the gravitational field in the space is given as $\left(-\frac{K}{r^2}\right)$. Taking the reference point to be at r = 2 cm with gravitational potential V = 10 J/kg. Find the gravitational potential at r = 3 cm in SI unit (Given, that K = 6Jcm/kg)
(1) 9 (2) 10 (3) 11 (4) 12

Sol. **(3)**

$\Delta V = -\int_2^3 \vec{E}.d\vec{r}$

$V(3) - V(2) = -\int_2^3 \frac{-K}{r^2}.dr$

$V(3) - 10 = -K\left(\frac{1}{r}\right)_2^3$

$V(3) - 10 = -6\left[\frac{1}{3} - \frac{1}{2}\right]$

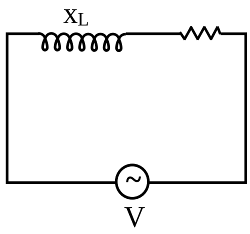
$V - 10 = -6\left[\frac{2-3}{6}\right] = 1$

$\boxed{V = 11}$

10. In a series LR circuit with $X_L = R$, power factor is P_1. If a capacitor of capacitance C with $X_C = X_L$ is added to the circuit the power factor becomes P_2. The ratio of P_1 to P_2 will be :

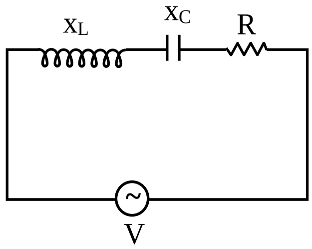
(1) 1:3 (2) 1: 2 (3) $1:\sqrt{2}$ (4) 1: 1

Sol. **(3)**



Power factor = $\cos\phi = \frac{R}{Z}$

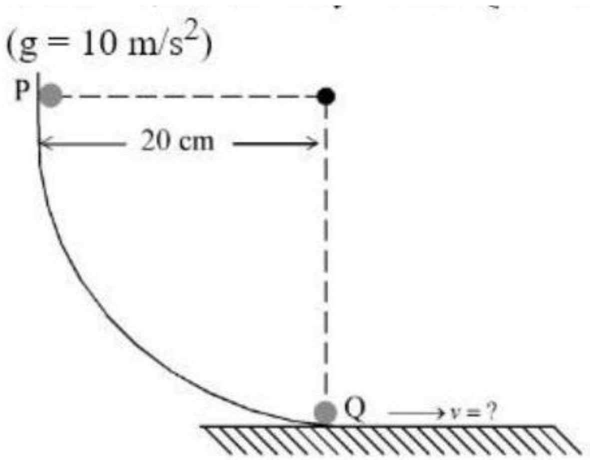
$$P_1 = \frac{R}{\sqrt{X_L^2 + R^2}} = \frac{R}{\sqrt{R^2 + R^2}} = \frac{R}{\sqrt{2}R} = \frac{1}{\sqrt{2}}$$



$$P_2 = \frac{R}{Z} = \frac{R}{\sqrt{(X_L - X_C)^2 + R^2}} = \frac{R}{R} = 1$$

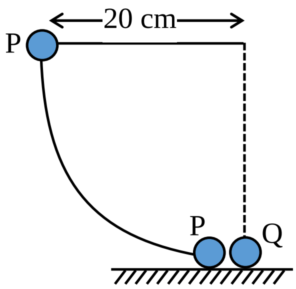
$$\frac{P_1}{P_2} = \frac{1}{\sqrt{2}}$$

11. As per the given figure, a small ball P slides down the quadrant of a circle and hits the other ball Q of equal mass which is initially at rest. Neglecting the effect of friction and assume the collision to be elastic, the velocity of ball Q after collision will be :



(1) 0 (2) 4 m/s (3) 2 m/s (4) 0.25 m/s

Sol. **(3)**



Energy conservation for ‘P’

$$mgh = \frac{1}{2}mV^2$$

$$V = \sqrt{2gh}$$

$$V = \sqrt{2 \times 10 \times 0.2}$$

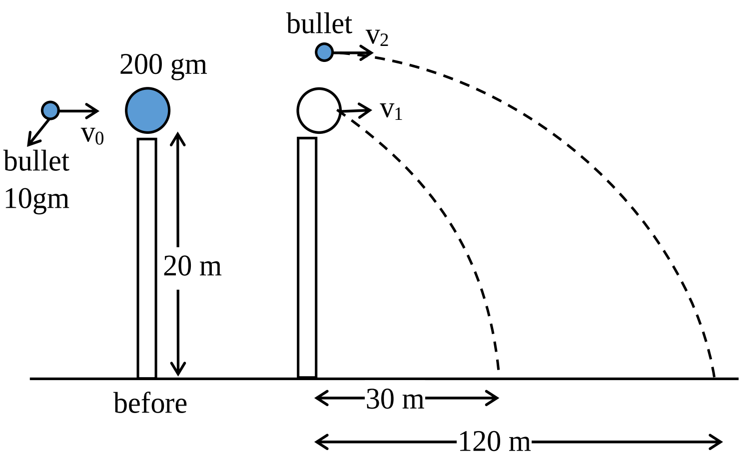
$$\boxed{V = 2m/sec}$$

Now collision between P and Q is elastic and both have same mass then P will transfer all velocity to then Q. So velocity Q will be 2 m/sec

12. A ball of mass 200 g rests on a vertical post of height 20 m. A bullet of mass 10 g, travelling in horizontal direction, hits the centre of the ball. After collision both travels independently. The ball hits the ground at a distance 30 m and the bullet at a distance of 120 m from the foot of the post. The value of initial velocity of the bullet will be (if $g = 10\ m/s^2$) :

(1)360 m/s (2) 400 m/s (3) 60 m/s (4) 120 m/s

Sol. **(1)**



Time to reach ground will be same for both

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 20}{10}} = 2sec$$

Range of bullet = 120

$$120 = v_2(2) \Rightarrow \boxed{v_2 = 60m/sec}$$

Range of ball = 30

$$30 = V_1(2) \Rightarrow \boxed{v_1 = 15 m/sec}$$

Now apply momentum conservation

$$P_i = P_f$$

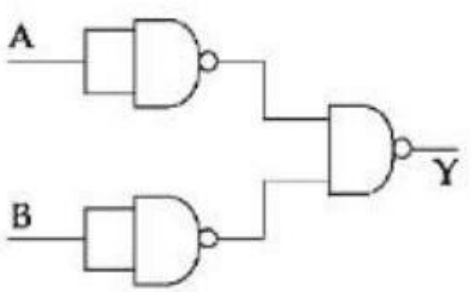
$$P_{ball} + P_{bullet} = P_{ball} + P_{bullet}$$

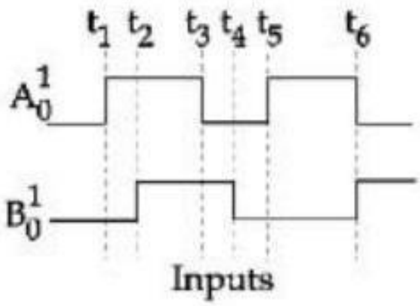
$$0 + \left(\frac{10}{1000}\right)v_0 = \left(\frac{200}{1000}\right)(15) + \left(\frac{10}{1000} \times 60\right)$$

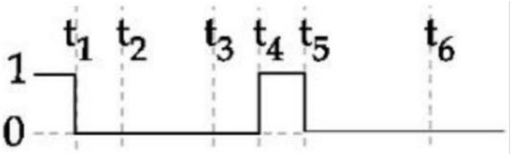
$$10v_0 = 3000 + 600$$

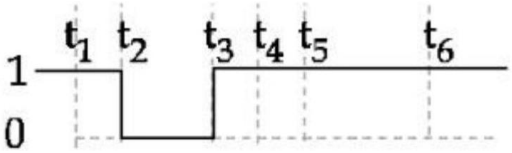
$$v_0 = \frac{3600}{10} \Rightarrow \boxed{v_0 = 360 m/sec}$$

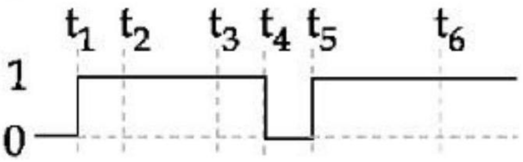
13. The output waveform of the given logical circuit for the following inputs A and B as shown below, is

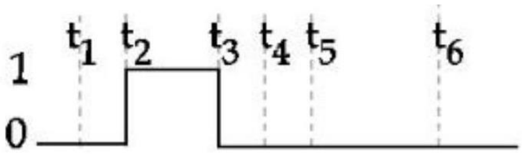




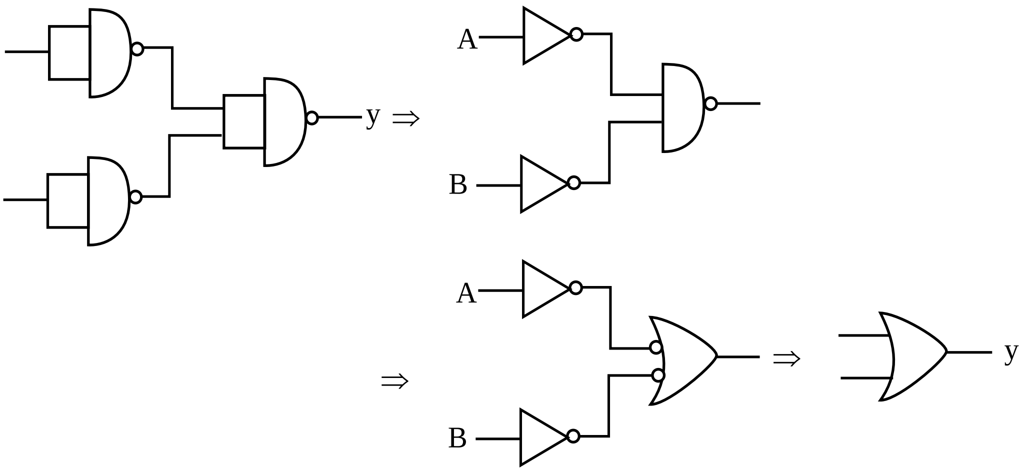
(1)


(2)


(3)


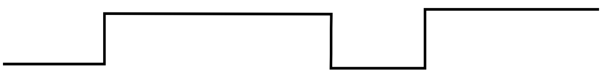
(4)


Sol. **(3)**



y = A + B

A	0	1	1	0	0	1	0
B	0	0	1	1	0	0	1
y	0	1	1	1	0	1	1



14. The charge flowing in a conductor changes with time as $Q(t) = \alpha t - \beta t^2 + \gamma t^3$. Where α, β and γ are constants. Minimum value of current is :

(1) $\alpha - \frac{3\beta^2}{\gamma}$ (2) $\alpha - \frac{\gamma^2}{3\beta}$ (3) $\alpha - \frac{\beta^2}{3\gamma}$ (4) $\beta - \frac{\alpha^2}{3\gamma}$

Sol. **(3)**

$Q = \alpha t - \beta t^2 + \gamma t^3$

$I = \frac{dQ}{dt} = \alpha - 2\beta t + 3\gamma t^2$

$\frac{dI}{dt} = 0 = 0 - 2\beta + 6\gamma t \Rightarrow t = \frac{2\beta}{6\gamma} = \frac{\beta}{2\gamma}$

$I_{min} = \alpha - 2\beta\left(\frac{\beta}{3\gamma}\right) + 3\gamma\left(\frac{\beta}{3\gamma}\right)^2$

$= \alpha - \frac{2\beta^2}{3\gamma} + \frac{\beta^2}{3\gamma}$

$\boxed{I_{min} = \alpha - \frac{\beta^2}{3\gamma}}$

15. Choose the correct relationship between Poisson ratio (σ), bulk modulus (K) and modulus of rigidity (η) of a given solid object :

(1) $\sigma = \frac{3K + 2\eta}{6K + 2\eta}$ (2) $\sigma = \frac{3K - 2\eta}{6K + 2\eta}$ (3) $\sigma = \frac{6K + 2\eta}{3K - 2\eta}$ (4) $\sigma = \frac{6K - 2\eta}{3K - 2\eta}$

Sol. **(2)**

$Y = 2\eta[1+\sigma]$

and $Y = 3K[1-2\sigma]$

Now $2\eta(1+\sigma) = 3K(1-2\sigma)$

$2\eta\sigma + 2\eta = 3K - 6K\sigma$

$(2n + 6K)\sigma = 3K - 2n$

$$\boxed{\sigma = \frac{3K - 2\eta}{2\eta + 6K}}$$

16. Speed of an electron in Bohr's 7^{th} orbit for Hydrogen atom is 3.6×10^6 m/s. The corresponding speed of the electron in 3^{rd} orbit, in m/s is:

(1) (1.8×10^6) (2) (3.6×10^6) (3) (7.5×10^6) (4) (8.4×10^6)

Sol. **(4)**

We now

$V \propto \frac{z}{n}$

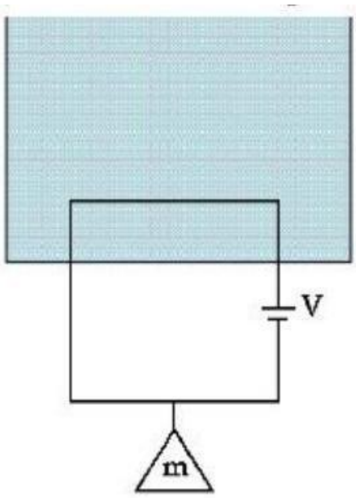
$\frac{V_3}{V_7} = \frac{7}{3}$

$V_3 = V_7 \times \frac{7}{3} = 3.6 \times 10^6 \times \frac{7}{3} = 1.2 \times 7 \times 10^6$

$$\boxed{V_3 = 8.4 \times 10^6 \, m/s}$$

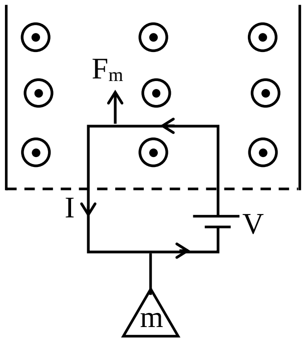
17. A massless square loop, of wire of resistance 10Ω, supporting a mass of 1 g, hangs vertically with one of its sides in a uniform magnetic field of 10^3G, directed outwards in the shaded region. A dc voltage V is applied to the loop. For what value of V, the magnetic force will exactly balance the weight of the supporting mass of 1 g ?

(If sides of the loop $= 10$ cm, $g = 10$ ms^{-2})



(1) $\frac{1}{10}$ V (2) 100 V (3) 10 V (4) 1 V

Sol. **(3)**



For balancing $\rightarrow$ $l = 10$ cm ,

$B = 10^3$ G $= 0.1$ T,

$m = 1$ g

$F_m = mg$

$I\ell B = mg$

$$\frac{V}{R}(0.1)(0.1) = \frac{1}{1000} \times 10$$

$$\frac{V}{100R} = \frac{1}{100}$$

$$\frac{V}{10} = 1 \Rightarrow \boxed{V = 10\text{Volt}}$$

18. Electric field in a certain region is given by $\vec{E} = \left(\frac{A}{x^2}\hat{i} + \frac{B}{y^2}\hat{j}\right)$. The SI unit of A and B are :

(1) Nm^3C^{-1}; Nm^2C^{-1} (2) Nm^2C^{-1}; Nm^3C^{-1} (3) Nm^3C; Nm^2C (4) Nm^2C; Nm^3C

Sol. **(2)**

$$\vec{E} = \frac{A}{x^2}\hat{i} + \frac{B}{y^3}\hat{j}$$

Unit of A $\rightarrow$ $\frac{N}{c} \times m^2 = Nm^2c^{-1}$

Unit of B $\rightarrow$ $\frac{N}{c} \times m^3 = Nm^3c^{-1}$

19. The height of liquid column raised in a capillary tube of certain radius when dipped in liquid A vertically is, 5 cm. If the tube is dipped in a similar manner in another liquid B of surface tension and density double the values of liquid A, the height of liquid column raised in liquid B would be m

(1) 0.05 (2) 0.10 (3) 0.20 (4) 0.5

Sol. **(1)**

$$h = \frac{2T\cos\theta}{r\rho g}$$

$$h \propto \frac{T}{\rho}$$

$$\frac{h_2}{h_1} = \frac{T_2}{T_1} \times \frac{\rho_1}{\rho_2}$$

$$\frac{h_2}{5cm} = \frac{2T}{T} \times \frac{\rho}{2\rho} = 1$$

$$\boxed{h_2 = 5cm = 0.05m}$$

20. A sinusoidal carrier voltage is amplitude modulated. The resultant amplitude modulated wave has maximum and minimum amplitude of 120 V and 80 V respectively. The amplitude of each sideband is :

(1) 20 V (2) 15 V (3) 10 V (4) 5 V

Sol. **(3)**

$V_{max} = V_m + V_c$

$120 = V_c + V_m$ ...(1)

$V_{min} = V_c - V_m$

$80 = V_c - V_m$ ...(2)

(1) + (2)

$200 = 2V_c \Rightarrow \boxed{V_c = 100}$

$V_M = 120 - 100 = 20 \Rightarrow \boxed{V_M = 20}$

$\mu = \frac{V_m}{V_c} = \frac{20}{100} = 0.2$

Amplitude of side bond = $\frac{\mu A_c}{2} = 0.2 \times \frac{100}{2} = 10V$

SECTION - B

21. The general displacement of a simple harmonic oscillator is $x = A\sin\omega t$. Let T be its time period. The slope of its potential energy (U) - time (t) curve will be maximum when $t = \frac{T}{\beta}$. The value of β is

Sol. **(8)**

$x = A\sin(\omega t)$

Potential energy $U = \frac{1}{2}kx^2$

$U = \frac{1}{2}.K.A^2\sin^2(\omega t)$

$\frac{dU}{dt} = \frac{KA^2}{2}.2\sin(\omega t)\cos(\omega t).\omega$

Slope $= \frac{dU}{dt} = \frac{\omega KA^2}{2}\sin(2\omega t)$

$\rightarrow$ Slope will be maximum for $\sin(2\omega t)$ will maximum

$2\omega t = \frac{\pi}{2}$

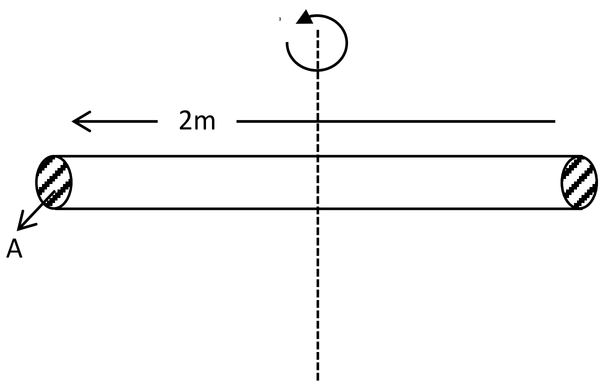
$2\omega.\frac{T}{\beta} = \frac{\pi}{2}$

$2\frac{2\pi}{T} \times \frac{T}{\beta} = \frac{\pi}{2} \Rightarrow \beta = 8$

Ans. = 8

22. A thin uniform rod of length 2 m, cross sectional area ' A ' and density ' d ' is rotated about an axis passing through the centre and perpendicular to its length with angular velocity ω. If value of ω in terms of its rotational kinetic energy E is $\sqrt{\frac{\alpha E}{Ad}}$ then value of α is

Sol. **(3)**



density $=\mathrm{d}$

Area = A

mass $m=d.A\ell$

$m=dA(2)=2Ad$ .... (1)

$$K.E.=\frac{1}{2}I\omega^2$$

$$E=\frac{1}{2}.\frac{m\ell^2}{12}.\omega^2$$

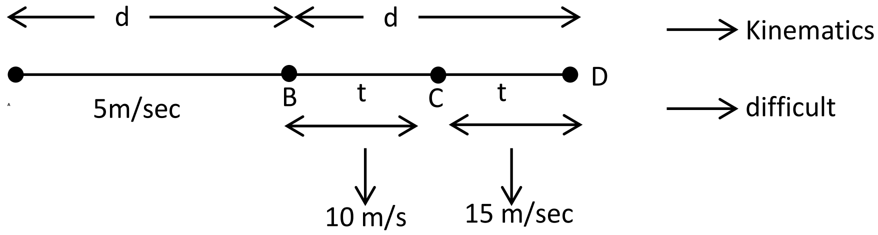
$$E=\frac{1}{24}.2Ad.(2)^2\omega^2$$

$$\frac{24E}{8Ad}=\omega^2 \Rightarrow \omega=\sqrt{\frac{3E}{Ad}}$$

Ans. $\alpha=3$

23. A horse rider covers half the distance with 5 m/s speed. The remaining part of the distance was travelled with speed 10 m/s for half the time and with speed 15 m/s for other half of the time. The mean speed of the rider averaged over the whole time of motion is $\frac{x}{7}$ m/s. The value of x is

Sol. **(50)**



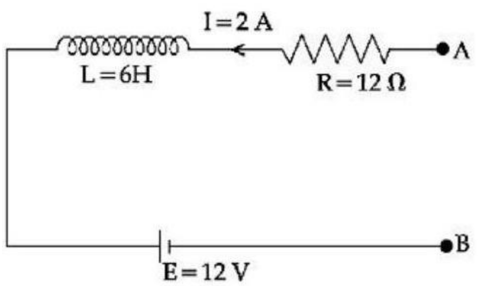
Avg. speed from B to D $\rightarrow V_{BD} = \frac{10+15}{2} = \frac{25}{2} m/sec$

Now, $\frac{2}{V_{ay}} = \frac{1}{5} + \frac{2}{25}$

$\frac{2}{V_{ag}} = \frac{7}{25} \Rightarrow V_{ag} = \frac{50}{7}$

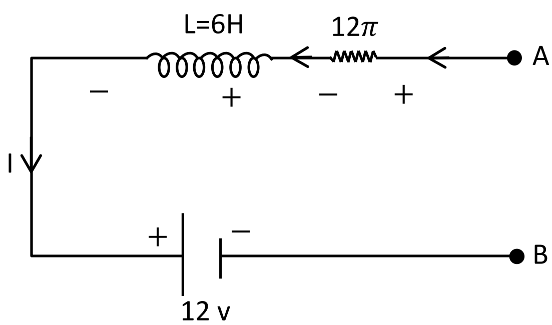
Ans. $x = 50$

24.



As per the given figure, if $\frac{dI}{dt} = -1\ A/s$ then the value of V_{AB} at this instant will be V.

Sol. **(30)**



$I = 2A$

$\frac{dI}{dt} = -1A/\sec$

$V_A - IR - L\frac{dI}{dt} - 12 = V_B$

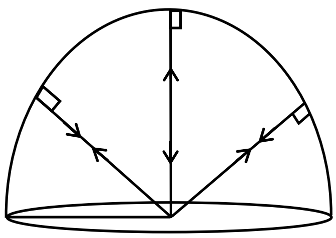
$V_A - 2(12) + 6(1) - 12 = V_B$

$V_A - V_B = 24 + 12 - 6 = 24 + 6 = 30$

Ans. 30

25. A point source of light is placed at the centre of curvature of a hemispherical surface. The source emits a power of 24 W. The radius of curvature of hemisphere is 10 cm and the inner surface is completely reflecting. The force on the hemisphere due to the light falling on it is ______ 10^{-8} N

Sol. **(4)**



$\theta = 0°$

Presses due reflecting surface $= \frac{2I}{C}$

Net force $= \frac{2I}{C}$ Area .... (1)

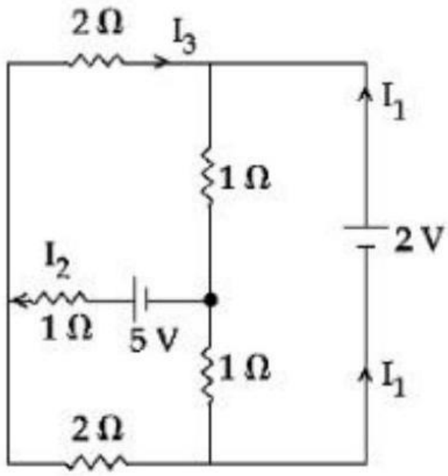
Now $I = \frac{\text{Power}}{\text{Area}} = \frac{\text{Power}}{4\pi r^2}$

From $F_{net} = \frac{2I}{C} \times$ Projected Area

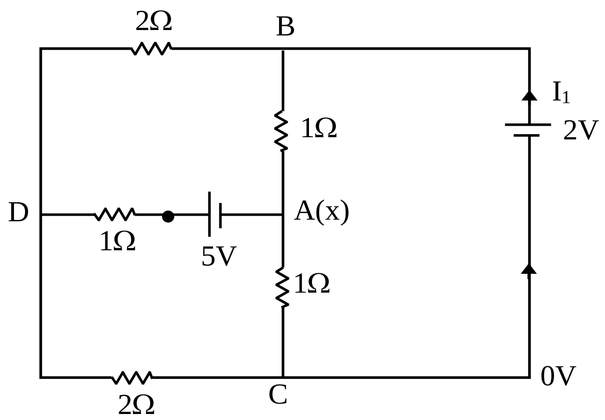
$F_{net} = \frac{2}{C} \times \frac{\text{Power}}{4\pi r^2} \times \pi r^2$

$F_{net} = \frac{2 \times 24}{3 \times 10^8 \times 4} = 4 \times 10^{-8}$

26. In the following circuit, the magnitude of current I_1, is _______ A.



Sol. **(1.5)**



Let at junction A → voltage = x

$V_A = x$

$V_D = y$

$V_C = 0$

$V_B = 2$

At junction 'A'

$$\frac{x-2}{1}+\frac{x-0}{1}+\frac{x+5-y}{1}=0$$

$3x - y + 3 = 0$ ...(1)

At junction 'D'

$$\frac{y-0}{2}+\frac{y-2}{2}+\frac{y-x-5}{1}=0$$

$4y - 2x = 12$

$2y - x = 6$ ...(2)

From (1) and (2)

$x = 0$; $y = 3$

So curent through 2V cell is

$$I = \frac{3}{2} = 1.5 \text{ A}$$

27. In a screw gauge, there are 100 divisions on the circular scale and the main scale moves by 0.5 mm on a complete rotation of the circular scale. The zero of circular scale lies 6 divisions below the line of graduation when two studs are brought in contact with each other. When a wire is placed between the studs, 4 linear scale divisions are clearly visible while 46th division the circular scale coincide with the reference line. The diameter of the wire is ________ $\times 10^{-2}$ mm

Sol. **(220)**

Pitch = 0.5 mm

$$\text{L.C.} = \frac{\text{pitch}}{\text{circular division}} = \frac{0.5\text{mm}}{100} = 0.005\text{mm}$$

Zero error = 6 × L.C. = 6 × (0.005) mm

Reading = main linear scale reading + n(L.C.) – zero error

= 4(0.5mm) + 46 (0.005) – 6(0.005)

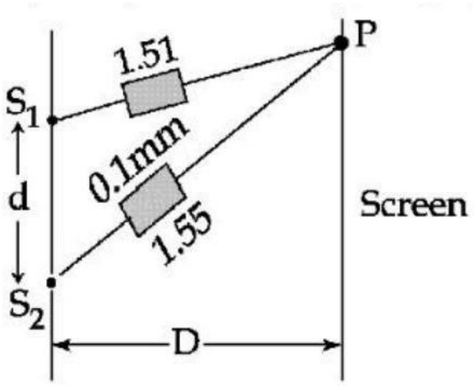
= 2 mm + 40 × 0.005 mm

$= 2\text{ mm} + \frac{200}{1000}\text{ mm}$

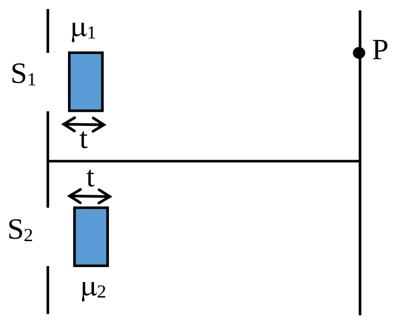
= 2.2 mm

Rading = 220×10^{-2} mm

28. In Young's double slit experiment, two slits S_1 and S_2 are ' d ' distance apart and the separation from slits to screen is D (as shown in figure). Now if two transparent slabs of equal thickness 0.1 mm but refractive index 1.51 and 1.55 are introduced in the path of beam ($\lambda = 4000\text{Å}$) from S_1 and S_2 respectively. The central bright fringe spot will shift by number of fringes.



Sol. **(10)**



$\mu_1 = 1.51$ $\quad t = 0.1\text{mm}$

$\mu_2 = 1.55$ $\quad \lambda = 4000\text{Å}$

Shifting central maxima

$$\Delta x = [S_1P + (\mu_1 - 1)t] - [S_2P + (\mu_2 - 1)t]$$

$$0 = (S_1P - S_2P) + (\mu_1 - 1)t - (\mu_2 - 1)t$$

$$0 = \frac{yd}{D} + (\mu_1 - \mu_2)t$$

$$(\mu_2 - \mu_1)t = \frac{yd}{D}$$

$$(1.55 - 1.51)(0.1\text{mm}) = y \times \frac{d}{D}$$

$$\frac{D}{d}(0.04 \times 0.1) \times 10^{-3} = y \qquad \ldots(1)$$

Now

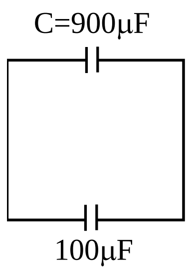
Fringe width $\Rightarrow \beta = \frac{\lambda D}{d}$

No. of fringes shifted $= \frac{y}{\beta} = \frac{4 \times 10^{-6}}{4000\text{Å}} = 10$

Ans. 10

29. A capacitor of capacitance $900\mu F$ is charged by a 100 V battery. The capacitor is disconnected from the battery and connected to another uncharged identical capacitor such that one plate of uncharged capacitor connected to positive plate and another plate of uncharged capacitor connected to negative plate of the charged capacitor. The loss of energy in this process is measured as $x \times 10^{-2}$ J. The value of x is

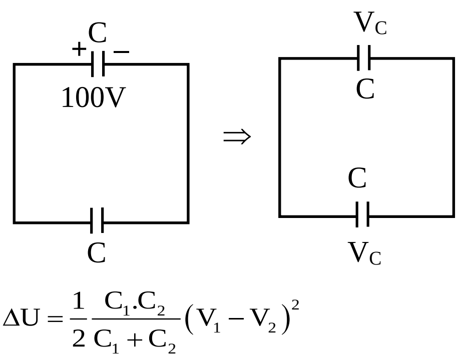
Sol. **(225)**



$Q = 900 \times 100\ \mu C$

$Q = 9 \times 10^{-2}$ C $\qquad \ldots(1)$

Now



$$\Delta U = \frac{1}{2}\frac{C_1.C_2}{C_1 + C_2}(V_1 - V_2)^2$$

$$= \frac{1}{2} \times \frac{C \times C}{2C} \times (100 - 0)^2$$

$$=\frac{C}{4}\times 100\times 100$$

$$=\frac{900}{4}\times 10^{-6}\times 10^{4}$$

$$=\frac{9}{4}=2.25\text{J}$$

$$\boxed{\Delta U=225\times 10^{-2}\,\text{J}}$$

30. In an experiment for estimating the value of focal length of converging mirror, image of an object placed at 40 cm from the pole of the mirror is formed at distance 120 cm from the pole of the mirror. These distances are measured with a modified scale in which there are 20 small divisions in 1 cm. The value of error in measurement of focal length of the mirror is $\frac{1}{K}$ cm. The value of K is

Sol. **32**

$$\frac{1}{f}=\frac{1}{v}+\frac{1}{u} \qquad dv=du=\frac{1\text{cm}}{20}=0.05\text{cm} \ \ (\text{given})$$

$$f^{-1}=v^{-1}+u^{-1}$$

$$(-1)f^{-2}df=(-1)v^{-2}dv-u^{-2}du$$

$$\frac{df}{f^{2}}=\frac{dv}{v^{2}}+\frac{du}{u^{2}} \qquad \ldots(1)$$

$$\frac{1}{f}=\frac{1}{(-120)}+\frac{1}{-40}$$

$$\frac{1}{f}=\frac{1+3}{(-120)}=\frac{4}{-120} \Rightarrow \boxed{f=-30\text{cm}}$$

Put value of f, du, dv in (1)

$$\frac{df}{(30)^{2}}=\frac{0.05}{(120)^{2}}+\frac{0.05}{(40)^{2}}$$

$$df=\frac{1}{32}\text{cm} \qquad \text{so} \qquad \boxed{K=32}$$

Chemistry

SECTION - A

31. Lithium aluminium hydride can be prepared from the reaction of

(1) LiH and $Al(OH)_3$ (2) LiH and Al_2Cl_6

(3) LiCl and Al_2H_6 (4) LiCl, Al and H_2

Sol. 2

$8\ LiH + Al_2Cl_6 \rightarrow 2\ LiAlH_4 + 6\ LiCl$

32. Amongst the following compounds, which one is an antacid?

(1) Terfenadine (2) Meprobamate (3) Brompheniramine (4) Ranitidine

Sol. **4**

Ranitidine is an antacid it is an antihistamine and decrease the reaction of gastric juice in stomach

33. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : In expensive scientific instruments, silica gel is kept in watch-glasses or in semipermeable membrane bags.

Reason (R) : Silica gel adsorbs moisture from air via adsorption, thus protects the instrument from water corrosion (rusting) and / or prevents malfunctioning.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(2) (A) is false but (R) is true

(3) Both (A) and (R) are true and (R) is the correct explanation of (A)

(4) (A) is true but (R) is false

Sol. **3**

Theory based

34. Match List I with List II

LIST I (Atomic number)		LIST II (Block of periodic table)	
A.	37	I.	p-block
B.	78	II.	d-block
C.	52	III.	f-block
D.	65	IV.	s-block

Choose the correct answer from the options given below:

(1) A - IV, B - III, C - II, D – I (2) A - II, B - IV, C - I, D – III

(3) A - IV, B - II, C - I, D – III (4) A - I, B - III, C - IV, D - II

Sol. **3**

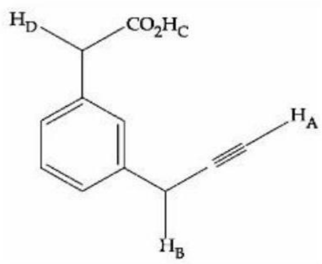
37 (K) s-block

78 (pt) d-block

52 (Te) p-block

65 (Tb) f-block

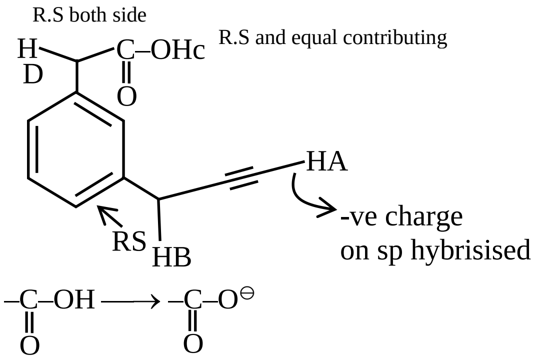
35. What is the correct order of acidity of the protons marked A-D in the given compounds ?



(1) $H_C > H_A > H_D > H_B$ (2) $H_D > H_C > H_B > H_A$

(3) $H_C > H_D > H_B > H_A$ (4) $H_C > H_D > H_A > H_B$

Sol. **4**



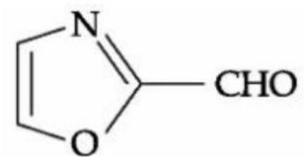
Equal contributing and resonance stablize

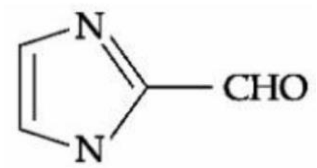
So order $H_C > H_D > H_A > H_B$

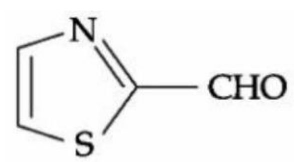
36. Which of the following compounds would give the following set of qualitative analysis?

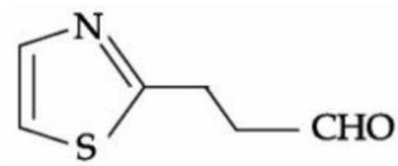
(i) Fehling's Test : Positive

(ii) Na fusion extract upon treatment with sodium nitroprusside gives a blood red colour but not prussian blue.

(1) 

(2) 

(3) 

(4) 

Sol. **4**

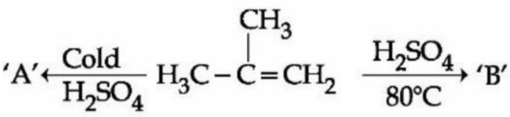
fehling test gives positive result for aliphatic aldehyde While sodium nitroprasside gives blood red color with S and N.

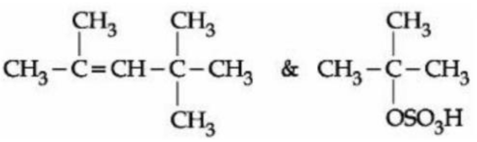
So Na+N+C+S → NaSCN (Sodium thiocyanate)

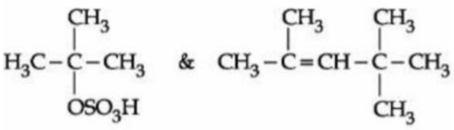
$SCN^- + Fe^{3+} \rightarrow [Fe(SCN)]^{2+}$ Ferric thiocyanate (Blood red color)

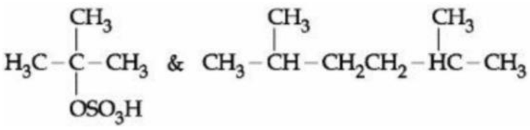
Confims presence of N and S

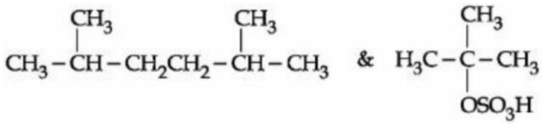
37. The major products ' A' and ' B ', respectively, are



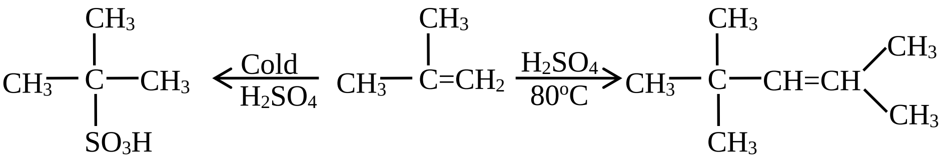
(1) 

(2) 

(3) 

(4) 

Sol. **2**



38. During the qualitative analysis of SO_3^{2-} using dilute H_2SO_4, SO_2 gas is evolved which turns $K_2Cr_2O_7$ solution (acidified with dilute H_2SO_4) :

(1) green (2) blue (3) red (4) black

Sol. **1**

$Na_2SO_3 + HCl \rightarrow NaCl + H_2O + SO_2\uparrow$

$K_2Cr_2O_7 + H_2SO_4 + SO_2 \rightarrow K_2SO_4 + \underset{\text{green}}{Cr_2(SO_4)_3} + H_2O$

39. In the wet tests for identification of various cations by precipitation, which transition element cation doesn't belong to group IV in qualitative inorganic analysis ?

(1) Ni^{2+} (2) Zn^{2+} (3) Co^{2+} (4) Fe^{3+}

sol. **4**

Zn^{+2}, CO^{+2}, Ni^{+2}, IVth group

Fe^{+3} = IIIrd group

40. For OF_2 molecule consider the following :

A. Number of lone pairs on oxygen is 2.

B. FOF angle is less than 104.5°.

C. Oxidation state of O is −2.

D. Molecule is bent ' V ' shaped.

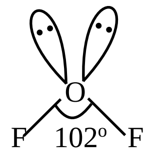
E. Molecular geometry is linear.

correct options are:

(1) A, C, D only (2) C, D, E only (3) A, B, D only (4) B, E, A only

Sol. **3**

OF_2

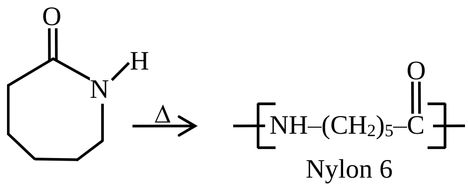


2 l.pe$^-$ in 'O'
bond angle 102°
bent/V shape

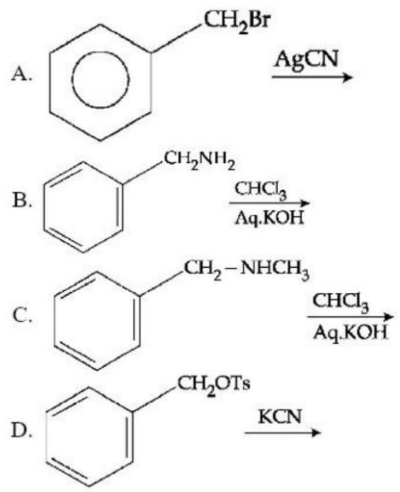
41. Caprolactam when heated at high temperature in presence of water, gives

(1) Nylon 6, 6 (2) Nylon 6 (3) Teflon (4) Dacron

Sol. **2**



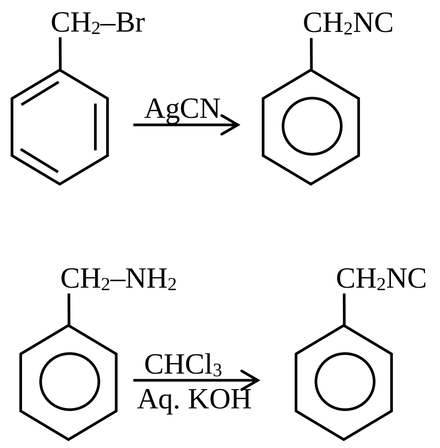
42. Benzyl isocyanide can be obtained by :



Choose the correct answer from the options given below :

(1) A and D (2) Only B (3) B and C (4) A and B

Sol. **4**



43. Formation of photochemical smog involves the following reaction in which A, B and C are respectively.

$$\text{i. } NO_2 \xrightarrow{h\nu} A + B$$
$$\text{ii. } B + O_2 \rightarrow C$$
$$\text{iii. } A + C \rightarrow NO_2 + O_2$$

Choose the correct answer from the options given below:

(1) $O, N_2O \& NO$ (2) $O, NO \& NO_3^-$ (3) $NO, O \& O_3$ (4) $N, O_2 \& O_3$

Sol. **3**

$$NO_2 \xrightarrow{h\upsilon} \underset{(A)}{NO} + \underset{(B)}{O}$$

$$\underset{(B)}{O} \xrightarrow{O_2} \underset{(C)}{O_3}$$

44. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Ketoses give Seliwanoff's test faster than Aldoses.

Reason (R) : Ketoses undergo β-elimination followed by formation of furfural.

In the light of the above statements, choose the correct answer from the options given below :

(1) (A) is false but (R) is true

(2) (A) is true but (R) is false

(3) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(4) Both (A) and (R) are true and (R) is the correct explanation of (A)

Sol. **2**

Seliwanoff's test – Test to differentiate for ketose and aldose.

In this keto hexose are more rapidly dehydrated to form 5–hydroxy methyl furfural when heated in acidic medium which on condensation with resorcinol, as result brown red colored complex is formed.

45. Match List I with List II

LIST I (molecules/ions)		LIST II (No. of lone pairs of e^- on central atom)	
A.	IF_7	I.	Three
B.	ICl_4^-	II.	One
C.	XeF_6	III.	Two
D.	XeF_2	IV.	Zero

Choose the correct answer from the options given below:

(1) A - II, B - III, C - IV, D – I (2) A - II, B - I, C - IV, D – III

(3) A - IV, B - I, C - II, D – III (4) A - IV, B - III, C - II, D – I

Sol. **4**

Molecule	l.pe^- of C.M.
IF_7	0
ICl_4^-	2
XeF_6	1
XeF_2	3

46. To inhibit the growth of tumours, identify the compounds used from the following :

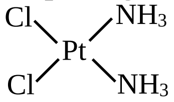
A. EDTA
B. Coordination Compounds of Pt
C. D – Penicillamine
D. Cis - Platin

Choose the correct answer from the option given below:

(1) B and D Only (2) C and D Only (3) A and C Only (4) A and B Only

Sol. **1**

Cis plating



is used as Anticancer agent

47. The alkaline earth metal sulphate(s) which are readily soluble in water is/are :

A. $BeSO_4$
B. $MgSO_4$
C. $CaSO_4$
D. $SrSO_4$
E. $BaSO_4$

Choose the correct answer from the options given below :

(1) B only (2) *A* and *B* (3) B and C (4) A only

Sol. **2**

$BeSO_4$ & $MgSO_4$ are soluble in water
$CaSO_4$ is partially soluble
$SrSO_4$ & $BaSO_4$ is insoluble

48. Which of the following is correct order of ligand field strength ?

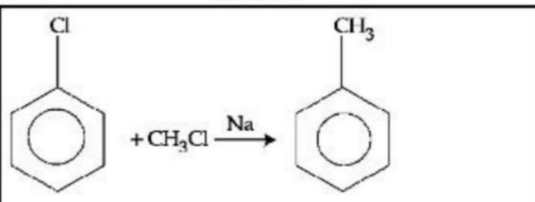
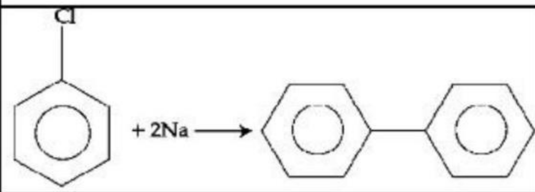
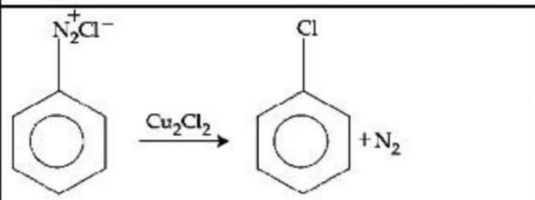
(1) $CO <$ en $< NH_3 < C_2O_4^{2-} < S^{2-}$
(2) $NH_3 <$ en $< CO < S^{2-} < C_2O_4^{2-}$
(3) $S^{2-} < C_2O_4^{2-} < NH_3 <$ en $< CO$
(4) $S^{2-} < NH_3 <$ en $< CO < C_2O_4^{2}$

Sol. **3**

order of ligand strength

$S^{2-} < C_2O_4^{2-} < NH_3 <$ en $< CO$

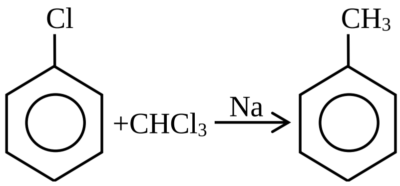
49. Match List I with List II

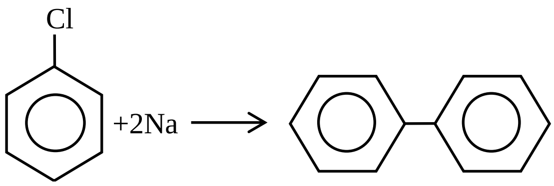
	LIST I		LIST II
A.		I.	Fittig reaction
B.		II.	Wurtz Fittig reaction
C.		III.	Finkelstein reaction
D.	$C_2H_5Cl + NaI \rightarrow C_2H_5I + NaCl$	IV.	Sandmeyer reaction

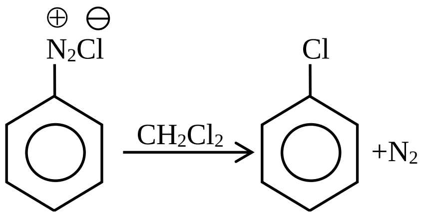
Choose the correct answer from the options given below:

(1) A - II, B - I, C - IV, D – III
(2) A - IV, B - II, C - III, D – I
(3) A - III, B - II, C - IV, D – I
(4) A - II, B - I, C - III, D – IV

Sol. **1**

(A)  Wurtz fittig reaction

(B)  fittig rxn

(C)  sandmeyer rxn

(D) $C_2H_5Cl + NaI \rightarrow C_2H_5I + NaCl$ Finkelstein rxn

50. In the extraction of copper, its sulphide ore is heated in a reverberatory furnace after mixing with silica to:

(1) remove FeO as $FeSiO_3$

(2) decrease the temperature needed for roasting of Cu_2S

(3) separate CuO as $CuSiO_3$

(4) remove calcium as $CaSiO_3$

Sol. **1**

The copper ore contains iron, it is mixed with silica before heating in reverberatory furnace, feO of slags off as $FeSiO_3$

$FeO + SiO_2 \rightarrow FeSiO_3$

SECTION - B

51. 600 mL of 0.01MHCl is mixed with 400 mL of $0.01MH_2SO_4$. The pH of the mixture is ______________ $\times 10^{-2}$. (Nearest integer)

[Given log2 2 = 0.30

$\log 3 = 0.48$

$\log 5 = 0.69$

$\log 7 = 0.84$

$\log 11 = 1.04$]

Sol. **186**

$$[H^+]_{mix} = \frac{(600 \times 0.01) + (400 \times 0.01 \times 2)}{1000}$$

$$= \frac{6+8}{1000} = 14 \times 10^{-3}$$

$pH = -\log(14 \times 10^{-3})$

$= 3 - \log 2 - \log 7$

$= 3 - 0.30 - 0.84$

$pH = 1.86$

52. The energy of one mole of photons of radiation of frequency 2×10^{12} Hz in J mol^{-1} is . (Nearest integer)

[Given : $h = 6.626 \times 10^{-34}$Js

$N_A = 6.022 \times 10^{23}\ mol^{-1}$]

Sol. **789**

$E_{photon}=6.626\times10^{-34}\times2\times10^{12}\times6.023\times10^{23}$

$= 79.81\times10$

$= 798.1 \approx 798$

53. Consider the cell

$Pt_{(s)}|H_2(g, 1\ atm)|H^+(aq, 1M)||Fe^{3+}(aq), Fe^{2+}(aq) \mid Pt(s)$

When the potential of the cell is 0.712 V at 298 K, the ratio $[Fe^{2+}]/[Fe^{3+}]$ is (Nearest integer)

Given : $Fe^{3+} + e^- = Fe^{2+}, E^{\theta} Fe^{3+}, Fe^{2+} \mid Pt = 0.771$

$$\frac{2.303RT}{F} = 0.06\ V$$

Sol. **10**

Cell reaction :-

$H_2+2Fe^{3+} \rightarrow 2H^++2Fe^{2+}$

$$E_{cell} = 0.771-\frac{2.303RT}{2F}\log\frac{[Fe^{2+}]^2[H^+]^2}{[Fe^{3+}]^2}$$

$0.712 = 0.771-0.03\ \log(x)^2$

$$\frac{0.059}{2}\log(x)^2 = 0.059$$

$\log x = 1$

$$x = \frac{[Fe^{2+}]}{[Fe^{3+}]} = 10$$

54. The number of electrons involved in the reduction of permanganate to manganese dioxide in acidic medium is

Sol. **3**

$4H^+ + MnO_4^- + 3e^- \rightarrow MnO_2 + 2H_2O$

55. A 300 mL bottle of soft drink has $0.2MCO_2$ dissolved in it. Assuming CO_2 behaves as an ideal gas, the volume of the dissolved CO_2 at STP is______mL. (Nearest integer)

Given : At STP, molar volume of an ideal gas is 22.7 L mol^{-1}

Sol. **1362**

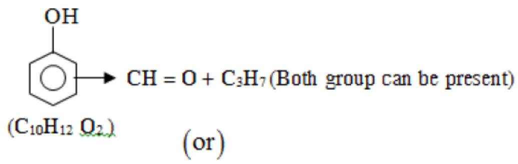
Mole of dissolved CO_2 = 0.2×300=60 mmol

$V_{CO_2} = 60\times10^{-3}\times22.7$

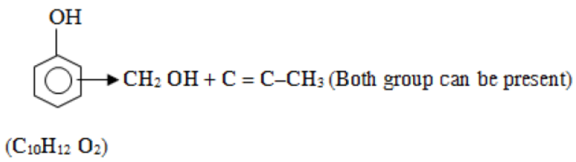
= 1362 ml

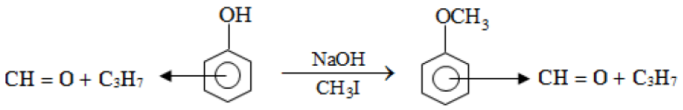
56. A trisubstituted compound ' A ', $C_{10}H_{12}O_2$ gives neutral $FeCl_3$ test positive. Treatment of compound 'A' with NaOH and CH_3Br gives $C_{11}H_{14}O_2$, with hydroiodic acid gives methyl iodide and with hot conc. NaOH gives a compound B, $C_{10}H_{12}O_2$. Compound 'A' also decolorises alkaline $KMnO_4$. The number of π bond/s present in the compound ' A ' is

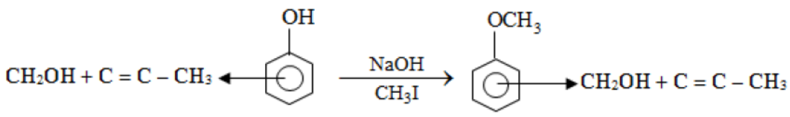
Sol. **4**



(or)







57. If compound A reacts with B following first order kinetics with rate constant $2.011 \times 10^{-3}\ s^{-1}$. The time taken by A (in seconds) to reduce from 7 g to 2 g will be (Nearest Integer)
[$\log 5 = 0.698, \log 7 = 0.845, \log 2 = 0.301$]

Sol. **623**

For Ist order:-

$$t = \frac{1}{2.011\times1^{-3}}\times2.303\times\log\frac{7}{2}$$

$$= \frac{2.303\times(0.845-0.301)}{2.011\times10^{-3}}$$

$= 622.9 \approx 623$

58. A solution containing 2 g of a non-volatile solute in 20 g of water boils at 373.52 K. The molecular mass of the solute is______________g mol^{-1}. (Nearest integer)
Given, water boils at 373 K, K_b for water $= 0.52\ K\ kg\ mol^{-1}$

Sol. **100**

$\Delta T_b = 373.52 - 373 = 0.52$

$\Delta T_b = iK_bm \qquad i=1$

$$0.52 = 0.52\times\frac{2/x}{20}\times1000$$

x = 100 gm/mol

59. When 2 litre of ideal gas expands isothermally into vacuum to a total volume of 6 litre, the change in internal energy is J. (Nearest integer)

Sol. **0**

$\Delta U = 0$

process is Isothermal

60. Some amount of dichloromethane (CH_2Cl_2) is added to 671.141 mL of chloroform ($CHCl_3$) to prepare 2.6×10^{-3}M solution of CH_2Cl_2(DCM). The concentration of DCM is ppm (by mass).

Given : atomic mass : C = 12

H = 1

Cl = 35.5

density of $CHCl_3 = 1.49 \text{ g cm}^{-3}$

Sol. **148.322**

Molar mass = 12+2+71

= 85

mmoles of DCM = $671.141 \times 2.6 \times 10^{-3}$

mass of solution = 1.49×671.141

$$\text{PPM} = \frac{671.141 \times 2.6 \times 10^{-3} \times 85 \times 10^{-3}}{1.49 \times 671.141} \times 10^{6}$$

148.322

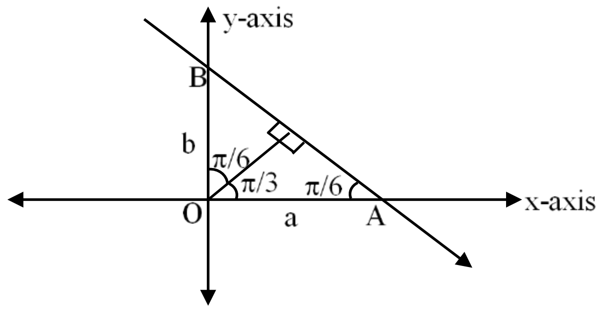
Mathematics

SECTION - A

61. A straight line cuts off the intercepts OA = a and OB = b on the positive directions of x-axis and y axis respectively. If the perpendicular from origin O to this line makes an angle of $\frac{\pi}{6}$ with positive direction of y-axis and the area of $\triangle$ OAB is $\frac{98}{3}\sqrt{3}$, then $a^2 - b^2$ is equal to:

(1) $\frac{392}{3}$ (2) $\frac{196}{3}$ (3) 98 (4) 196

Sol. **1**



In ΔAOB

$\tan\frac{\pi}{6} = \frac{OB}{OA} = \frac{b}{a}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{b}{a}$

$\Rightarrow \boxed{a = \sqrt{3}b}$

$\because$ area of triangle $\Delta OAB = \frac{1}{2} \times ab = \frac{98}{3} \times \sqrt{3}$

$\Rightarrow \frac{\sqrt{3}b^2}{2} = \frac{98}{\sqrt{3}}$

$\Rightarrow b^2 = \frac{98}{3} \times 2$

$\Rightarrow \boxed{b = \sqrt{\frac{196}{3}}}$

$\boxed{a = \sqrt{196}}$

$a^2 - b^2 = 196 - \frac{196}{3} = \frac{588 - 196}{3}$

$\Rightarrow \boxed{a^2 - b^2 = \frac{392}{3}}$

62. The minimum number of elements that must be added to the relation R={(a, b), (b, c)} on the set {a, b, c} so that is becomes symmetric and transitive is :

(1) 3 (2) 4 (3) 5 (4) 7

Sol. **4**

$R=\{(a,b),(b,c)\}$

For symmetric relation (b, a), (c, b) must be added in R

For transitive relation (a, c), (a, a), (b, b), (c, c), (c, a) must be added in R

So, minimum number of element = 7

63. If an unbiased die, marked with $-2,-1,0,1,2,3$ on its faces, is thrown five times, then the probability that the product of the outcomes is positive, is :

(1) $\frac{881}{2592}$ (2) $\frac{27}{288}$ (3) $\frac{440}{2592}$ (4) $\frac{521}{2592}$

Sol. **4**

Unbiased die. Marked with –2, –1, 0, 1, 2, 3

Product of outcomes is positive if

All time get positive number, 3 time positive and 2 time negative, 1 time positive and 4 time negative.

P (Product of the outcomes is positive) = $\underbrace{{}^5C_5\left(\frac{3}{6}\right)^5}_{\text{All positive}}+\underbrace{{}^5C_3\left(\frac{3}{6}\right)^3\left(\frac{2}{6}\right)^2}_{3\text{positive},2\text{negative}}+\underbrace{{}^5C_1\left(\frac{3}{6}\right)\left(\frac{2}{6}\right)^4}_{1\text{positive},4\text{negative}}$

$=\frac{3^5}{6^5}+\frac{10\times3^3\times2^2}{6^5}+\frac{5\times3\times2^4}{6^5}$

$=\frac{1563}{6^5}=\frac{521}{2592}$

64. If $\vec{a},\vec{b},\vec{c}$ are three non-zero vectors and $\hat{n}$ is a unit vector perpendicular to $\vec{c}$ such that $\vec{a}=\alpha\vec{b}-\hat{n}$, $(\alpha\neq0)$ and $\vec{b}\cdot\vec{c}=12$, then $|\vec{c}\times(\vec{a}\times\vec{b})|$ is equal to :

(1) 9 (2) 15 (3) 6 (4) 12

Sol. **4**

$\vec{a}=\alpha\vec{b}-\hat{n},\vec{b}.\vec{c}=12$

$\vec{c}\times(\vec{a}\times\vec{b})=(\vec{c}.\vec{b})\vec{a}-(\vec{c}.\vec{a})\vec{b}$

$\vec{c}\times(\vec{a}\times\vec{b})=12\vec{a}-(\vec{c}.\vec{a})\vec{b}$ ....(1)

$\because\ \vec{a}=\alpha\vec{b}-n$

$\vec{c}.\vec{a}=\alpha\vec{c}.\vec{b}-\vec{c}.n$

$\boxed{\vec{c}.\vec{a}=12\alpha}$ ....(2)

Equation (2) put in equation (1)

$\vec{c}\times(\vec{a}\times\vec{b})=12\vec{a}-12\alpha\vec{b}$

$\left|\vec{c}\times(\vec{a}\times\vec{b})\right|=12\left|\vec{a}-\alpha\vec{b}\right|$ $\left[\because\vec{a}-\alpha\vec{b}=-n\text{ then }|\vec{a}-\alpha\vec{b}|=1\right]$

$\Rightarrow\boxed{\left|\vec{c}\times(\vec{a}\times\vec{b})\right|=12}$

65. Among the statements :

(S1) $((p \vee q) \Rightarrow r) \Leftrightarrow (p \Rightarrow r)$

(S2) $((p \vee q) \Rightarrow r) \Leftrightarrow ((p \Rightarrow r) \vee (q \Rightarrow r))$

(1) only (S2) is a tautology (2) only (S1) is a tautology

(3) neither (S1) nor (S2) is a tautology (4) both (S1) and (S2) are tautologies

Sol. **3**

$S_1 : \left((p \vee q) \Rightarrow r\right) \Leftrightarrow (p \Rightarrow r)$

p	q	r	$(p \vee q) \Rightarrow r$ $(\sim p \wedge \sim q) \vee r$	$\sim p \vee r$	$\left((p \vee q) \Rightarrow r\right) \Leftrightarrow (p \Rightarrow r)$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	F	T	T	T
F	T	F	F	T	F
F	F	T	T	T	T

S_1 is not a tautology

$S_2 = \left((p \vee q) \Rightarrow r\right) \Leftrightarrow \left((p \Rightarrow r) \vee (q \Rightarrow r)\right)$

p	q	r	$(p \vee q) \Rightarrow r$	$(p \Rightarrow r) \vee (q \Rightarrow r)$	$\left((p \vee q) \Rightarrow r\right) \Leftrightarrow \left((p \Rightarrow r) \vee (q \Rightarrow r)\right)$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	F	F	T	T	T
F	T	F	F	T	F
F	F	T	T	T	T

S_2 is not a tautology

So, neither S_1 nor S_2 is a tautology.

66. If $P(h, k)$ be a point on the parabola $x = 4y^2$, which is nearest to the point $Q(0,33)$, then the distance of P from the directrix of the parabola $y^2 = 4(x + y)$ is equal to :

(1) 2 (2) 6 (3) 8 (4) 4

Sol. **2**

Equation of normal of the parabola $x = 4y^2$

At a point $P\left(\frac{t^2}{16}, \frac{2t}{16}\right)$ is

$$y + tx = \frac{2t}{16} + \frac{1}{16}t^3$$

$\because$ Normal pass through $Q(0,33)$ then

$$33 = \frac{t}{8} + \frac{t^3}{16}$$

$\Rightarrow t^3 + 2t - 528 = 0$

$\Rightarrow (t-8)(t^2 + 8 + 166) = 0$

$\Rightarrow t = 8$

Point P is (4, 1)

Given parabola is $y^2 = 4(x+y)$

$y^2 - 4y = 4x$

$(y-2)^2 = 4(x+1)$

directrix is x + 1 = – 1

$\boxed{x = -2}$

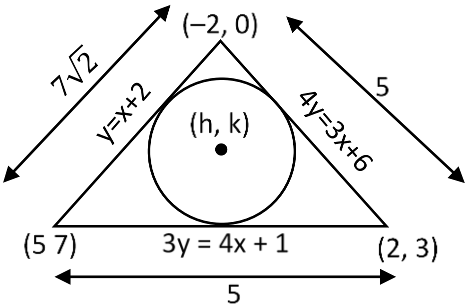
Distance of P(4, 1) from the directrix x = –2 is 6.

67. Let $y = x + 2, 4y = 3x + 6$ and $3y = 4x + 1$ be three tangent lines to the circle $(x-h)^2 + (y-k)^2 = r^2$.

Then $h + k$ is equal to :

(1) $5(1+\sqrt{2})$ (2) $5\sqrt{2}$ (3) 6 (4) 5

Sol. **4**



In centre of triangle is (h, k)

$$= \left(\frac{5(-2) + 2\times 7\sqrt{2} + 5\times 5}{5+5+7\sqrt{2}}, \frac{3\times(7\sqrt{2}) + 0\times 5 + 7\times 5}{5+5+7\sqrt{2}}\right)$$

$$= \left(\frac{14\sqrt{2}+15}{10+7\sqrt{2}}, \frac{21\sqrt{2}+35}{10+7\sqrt{2}}\right)$$

So, $h + k = \frac{14\sqrt{2}+15}{10+7\sqrt{2}} + \frac{21\sqrt{2}+35}{10+7\sqrt{2}}$

$$h + k = \frac{35\sqrt{2}+50}{7\sqrt{2}+10} = \frac{5(7\sqrt{2}+10)}{7\sqrt{2}+10} = 5$$

$\Rightarrow \boxed{h + k = 5}$

68. The number of points on the curve $y = 54x^5 - 135x^4 - 70x^3 + 180x^2 + 210x$ at which the normal lines are parallel to $x + 90y + 2 = 0$ is :

(1) 4 (2) 2 (3) 0 (4) 3

Sol. **1**

Given curve is $y = 54x^5 - 135x^4 - 70x^3 + 180x^2 + 210x$

$$\frac{dy}{dx} = 270x^4 - 540x^3 - 210x^2 + 360x + 210$$

$\because$ Normal is parallel to x + 90y + 2 = 0

Then tangent is $\perp^r$ to x + 90 y + 2 = 0

Then $(270x^4 - 540x^3 - 210x^2 + 360x + 210)\left(\frac{-1}{90}\right) = -1$

$$270x^4 - 540x^3 - 210x^2 + 360x + 120 = 0$$
$$\Rightarrow 9x^4 - 18x^3 - 7x^2 + 12x + 4 = 0$$
$$\Rightarrow (x-1)(x-2)(3x+1)(3x+2) = 0$$
$$\Rightarrow x = 1, 2, -\frac{1}{3}, -\frac{2}{3}$$

Number of points are 4

69. If $a_n = \frac{-2}{4n^2 - 16n + 15}$, then $a_1 + a_2 + \cdots ... + a_{25}$ is equal to:

(1) $\frac{52}{147}$ (2) $\frac{49}{138}$ (3) $\frac{50}{141}$ (4) $\frac{51}{144}$

Sol. **3**

given that $a_n = \frac{-2}{4n^2 - 16n + 15}$

$$a_1 + a_2 + a_3 + ...a_{25} = \sum_{n=1}^{25} \frac{-2}{(2n-3)(2n-5)}$$

$$= \sum_{n=1}^{25} \frac{(2n-5)-(2n-3)}{(2n-3)(2n-5)}$$

$$= \sum_{n=1}^{25} \left(\frac{1}{2n-3} - \frac{1}{(2n-5)}\right)$$

$$= \frac{1}{-1} - \frac{1}{-3}$$

$$+\frac{1}{1} - \frac{1}{-1}$$

$$+\frac{1}{3} - \frac{1}{1}$$

$$\vdots \qquad \vdots$$

$$\frac{1}{47} - \frac{1}{45}$$

$$= \frac{1}{47} + \frac{1}{3}$$

$$= \frac{3+47}{141} = \frac{50}{141}$$

70. If $\tan 15^\circ + \frac{1}{\tan 75^\circ} + \frac{1}{\tan 105^\circ} + \tan 195^\circ = 2a$, then the value of $\left(a + \frac{1}{a}\right)$ is :

(1) 2 (2) $4 - 2\sqrt{3}$ (3) $5 - \frac{3}{2}\sqrt{3}$ (4) 4

Sol. **4**

$\tan 15^\circ + \frac{1}{\tan 75^\circ} + \frac{1}{\tan 105^\circ} + \tan 195^\circ = 2a$

$\Rightarrow \tan 15^\circ + \frac{1}{\cot 15^\circ} - \frac{1}{\cot 15^\circ} + \tan 15^\circ = 2a$

$\Rightarrow \tan 15^\circ + \tan 15^\circ - \tan 15^\circ + \tan 15^\circ = 2a$

$\Rightarrow 2\tan 15^\circ = 2a$

$\Rightarrow a = \tan 15^\circ$

$a + \frac{1}{a} = \tan 15^\circ + \frac{1}{\tan 15^\circ}$

$= \tan 15^\circ + \cot 15^\circ$

$= 2 - \sqrt{3} + 2 + \sqrt{3}$

$\Rightarrow a + \frac{1}{a} = 4$

71. If the solution of the equation $\log_{\cos x} \cot x + 4\log_{\sin x} \tan x = 1, x \in \left(0, \frac{\pi}{2}\right)$, is $\sin^{-1}\left(\frac{\alpha + \sqrt{\beta}}{2}\right)$, where α and β are integers, then $\alpha + \beta$ is equal to :

(1) 5 (2) 6 (3) 4 (4) 3

Sol. **3**

$\log_{\cos x} \cot x + 4\log_{\sin x} \tan x = 1, x \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow \log_{\cos x} \frac{\cos x}{\sin x} + 4\log_{\sin x} \frac{\sin x}{\cos x} = 1$

$\Rightarrow 1 - \log_{\cos x} \sin x + 4 - 4\log_{\sin x} \cos x = 1$

$\Rightarrow 4 = \log_{\cos x} \sin x + 4\log_{\sin x} \cos x$

Let $\log_{\cos x} \sin x = t$

$\Rightarrow 4 = t + \frac{4}{t}$

$\Rightarrow t^2 - 4t + 4 = 0$

$\Rightarrow (t-2)^2 = 0$

$\Rightarrow t = 2$

$\Rightarrow \log_{\cos x} \sin x = 2$

$\Rightarrow \sin x = \cos^2 x$

$\Rightarrow \sin x = 1 - \sin^2 x$

$\Rightarrow \sin^2 x + \sin x - 1 = 0$

$\Rightarrow \sin x = \frac{-1 \pm \sqrt{1+4}}{2}$

$\Rightarrow \sin x = \frac{-1+\sqrt{5}}{2}, \frac{-1-\sqrt{5}}{2}$ $\quad \because \; x \in \left(0, \frac{\pi}{2}\right)$ then $\frac{-1-\sqrt{5}}{2}$ not possible

$\Rightarrow x = \sin^{-1}\left(\frac{-1+\sqrt{5}}{2}\right)$

$\because \; \alpha = -1, \beta = 5$ then

$\boxed{\alpha + \beta = 4}$

72. Let the system of linear equations

$x + y + kz = 2$

$2x + 3y - z = 1$

$3x + 4y + 2z = k$

have infinitely many solutions. Then the system

$(k + 1)x + (2k - 1)y = 7$

$(2k + 1)x + (k + 5)y = 10$

has:

(1) infinitely many solutions

(2) unique solution satisfying $x - y = 1$

(3) unique solution satisfying $x + y = 1$

(4) no solution

Sol. **3**

$x + y + kz = 2$

$2x + 3y – z = 1$

$3x + 4y + 2z = k$

Have Infinitely many solution then

$$\begin{vmatrix} 1 & 1 & k \\ 2 & 3 & -1 \\ 3 & 4 & 2 \end{vmatrix} = 0$$

$1(10) - 1(7) + k(8-9) = 0$

$\Rightarrow 10 – 7 – k = 0$

$\Rightarrow \boxed{k = 3}$

For k = 3

$4x + 5y = 7$

$7x + 8y = 10$

has unique solution and solution is (–2, 3).

Hence solution is unique and satisfying x + y = 1

73. The line l_1 passes through the point $(2,6,2)$ and is perpendicular to the plane $2x + y - 2z = 10$. Then the shortest distance between the line l_1 and the line $\frac{x+1}{2} = \frac{y+4}{-3} = \frac{z}{2}$ is :

(1) $\frac{13}{3}$ (2) $\frac{19}{3}$ (3) 7 (4) 9

Sol. **9**

equation of l_1 is $\frac{x-2}{2} = \frac{y-6}{1} = \frac{z-2}{-2}$

Let l_2 is $\frac{x+1}{2} = \frac{y+4}{-3} = \frac{z}{2}$

Point on l_1 is a = (2, 6, 2), direction $\vec{p} = <2,1,-2>$

Point on l_2 is b = (–1, –4, 0) direction $\vec{q} = <2,-3,2>$

Shortest distance between l_1 and l_2 = $\left|\frac{(\vec{a}-\vec{b}).(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right|$

$\because \quad \vec{p}\times\vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 2 & 1 & -2 \\ 2 & -3 & 2 \end{vmatrix} = \hat{i}(-4) - \hat{j}(8) + k(-8)$

$= \left|\frac{\langle 3,10,2\rangle \cdot \langle -4,-8,-8\rangle}{\sqrt{16+64+64}}\right|$

$= \left|\frac{-12-80-16}{\sqrt{144}}\right|$

$= \frac{108}{12}$

$= 9$

Shortest distance between the lines is 9.

74. Let $A = \begin{pmatrix} m & n \\ p & q \end{pmatrix}$, $d = |A| \neq 0$ and $|A - d(\text{Adj}A)| = 0$. Then

(1) $1 + d^2 = m^2 + q^2$ (2) $1 + d^2 = (m+q)^2$

(3) $(1+d)^2 = m^2 + q^2$ (4) $(1+d)^2 = (m+q)^2$

Sol. **4**

$A = \begin{bmatrix} m & n \\ p & q \end{bmatrix}$, d =|A| = mq – np

$A - d(\text{Adj. }A) = \begin{bmatrix} m & n \\ p & q \end{bmatrix} - d\begin{bmatrix} q & -n \\ -p & m \end{bmatrix}$

$= \begin{bmatrix} m - dq & n + dn \\ p + pd & q - dm \end{bmatrix}$

$|A - d(\text{Adj }A)| = (m - dq)(q - dm) - (n + dn)(p + pd) = 0$

$\Rightarrow mq - m^2d - dq^2 + d^2qm = np(1+d)^2$

$\Rightarrow (mq - m^2d - dq^2 + d^2qm) = (mq - d)(1 + d)^2$

$\Rightarrow mq - m^2d - dq^2 + d^2qm = mq + mqd^2 + 2mqd - d(1+d)^2$

$\Rightarrow d(1+d)^2 = m^2d + dq^2 + 2mqd$

$\Rightarrow \boxed{(1+d)^2 = (m+q)^2}$

75. If [t] denotes the greatest integer $\leq$ t, then the value of $\frac{3(e-1)}{e}\int_1^2 x^2 e^{[x]+[x^3]}dx$ is :

(1) $e^8 - 1$ (2) $e^7 - 1$ (3) $e^8 - e$ (4) $e^9 - e$

Sol. **3**

$$\frac{3(e-1)^2}{e}\int_1^2 x^2 e^{[x]+[x^3]}\,dx$$

$$\text{Let } I = \int_1^2 x^2 e^{[x]+[x^3]}\,dx$$

$$I = \int_1^2 x^2 e^{1+[x^3]}\,dx\,dx$$

$$\Rightarrow I = e\int_1^2 x^2 e^{[x^3]}\,dx$$

Let $x^3 = t$

$3x^2\,dx = dt$

$$I = \frac{e}{3}\int_1^8 e^{[t]}\,dt$$

$$\Rightarrow I = \frac{e}{3}\left[\int_1^2 e\,dt + \int_2^3 e^2 dt + \int_3^4 e^3 dt + \ldots + \int_7^8 e^7 dt\right]$$

$$\Rightarrow I = \frac{e}{3}\left[e + e^2 + e^3 + \ldots + e^7\right]$$

$$\Rightarrow I = \frac{e}{3}\left[\frac{e(e^7 - 1)}{e-1}\right]$$

$$\text{Therefore } \frac{3(e-1)}{e}\int_1^2 x^2 e^{[x]+[x^3]}\,dx = \frac{3(e-1)}{e}\times\frac{e^2}{3}\,\frac{(e^7-1)}{e-1}$$

$$\Rightarrow \frac{3(e-1)}{e}\int_1^2 x^2 e^{[x]+[x^3]}dx = e^8 - e$$

76. Let a unit vector $\widehat{OP}$ make angles α, β, γ with the positive directions of the co-ordinate axes OX, OY, OZ respectively, where $\beta \in \left(0, \frac{\pi}{2}\right)$. If $\widehat{OP}$ is perpendicular to the plane through points (1,2,3), (2,3,4) and (1,5,7), then which one of the following is true ?

(1) $\alpha \in \left(0, \frac{\pi}{2}\right)$ and $\gamma \in \left(0, \frac{\pi}{2}\right)$ (2) $\alpha \in \left(0, \frac{\pi}{2}\right)$ and $\gamma \in \left(\frac{\pi}{2}, \pi\right)$

(3) $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\gamma \in \left(\frac{\pi}{2}, \pi\right)$ (4) $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\gamma \in \left(0, \frac{\pi}{2}\right)$

Sol. **3**

$\because$ $\overrightarrow{OP}$ makes angle α, β, γ with positive directions of the co-ordinate axes then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

Point on planes are a(1, 2, 3), b(2, 3, 4) and c(1, 5, 7).

$\because\ \overrightarrow{ab} = \langle 1, 1, 1\rangle$

$\overrightarrow{ac} = \langle 0,3, 4\rangle$

normal vector of plane = $\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 0 & 3 & 4 \end{bmatrix}$

$= \hat{i}(1) - \hat{j}(4) + \hat{k}(3)$

$= \langle 1, -4, 3\rangle$

direction cosine of normal is $= \left\langle \pm\frac{1}{\sqrt{26}}, \pm\frac{4}{\sqrt{26}}, \pm\frac{3}{\sqrt{26}} \right\rangle$

then direction cosine of $\overrightarrow{op}$ is $\left\langle -\frac{1}{\sqrt{26}}, \frac{4}{\sqrt{26}}, -\frac{3}{\sqrt{26}} \right\rangle$

$\left(\because \beta \in \left(0, \frac{\pi}{2}\right)\right)$

Hence $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\gamma \in \left(\frac{\pi}{2}, \pi\right)$

77. The coefficient of x^{301} in $(1+x)^{500} + x(1+x)^{499} + x^2(1+x)^{498} + \cdots \ldots x^{500}$ is :

(1) ${}^{500}C_{300}$ (2) ${}^{501}C_{200}$ (3) ${}^{501}C_{302}$ (4) ${}^{500}C_{301}$

Sol. **2**

$x^0(1+x)^{500} + x(1+x)^{499} + x^2(1+x)^{498} + \ldots + x^{500}$

$= (1+x)^{500} \frac{\left[\left(\frac{x}{1+x}\right)^{501} - 1\right]}{\frac{x}{1+x} - 1}$

$= \frac{(1+x)^{500}(x^{501} - (1+x)^{501})}{(1+x)^{501}\left(\frac{-1}{x+x}\right)}$

$= (1+x)^{501} - x^{501}$

Coefficient of x^{301} in above expression is ${}^{501}C_{301}$ or ${}^{501}C_{200}$.

78. Let the solution curve $y = y(x)$ of the differential equation

$\frac{dy}{dx} - \frac{3x^5 \tan^{-1}(x^3)}{(1+x^6)^{\frac{3}{2}}} y = 2x \exp\left\{\frac{x^3 - \tan^{-1} x^3}{\sqrt{(1+x^6)}}\right\}$ pass through the origin. Then $y(1)$ is equal to :

(1) $\exp\left(\frac{4+\pi}{4\sqrt{2}}\right)$ (2) $\exp\left(\frac{1-\pi}{4\sqrt{2}}\right)$ (3) $\exp\left(\frac{\pi-4}{4\sqrt{2}}\right)$ (4) $\exp\left(\frac{4-\pi}{4\sqrt{2}}\right)$

Sol. **4**

$$\left(\frac{dy}{dx}\right) - \frac{3x^5 \tan^{-1}(x^3)}{(1+x^6)^{3/2}} y = 2x \exp\left(\frac{x^3 - \tan^{-1} x^3}{\sqrt{(1+x^6)}}\right)$$

above equation is linear differential equation.

$$\text{I.F.} = e^{\int \frac{-3x^5 \tan^{-1}(x^3)}{(1+x^6)^{3/2}} dx}$$

$$= e^{-\int \frac{3x^2 \cdot x^3 \tan^{-1}(x^3)}{(1+x^6)^{3/2}} dx}$$

Let $\tan^{-1}(x^3) = t$ then

$$\frac{3x^2 \cdot dx}{1+x^6} = dt$$

$$= e^{-\int \frac{t \tan t}{\sqrt{1+\tan^2 t}} dt}$$

$$= e^{-\int \frac{t \tan t}{\sec t} dt}$$

$$= e^{-\int t \sin t \, dt}$$

$$= e^{-[-t \cos t + \sin t]}$$

$$= e^{t \cos t - \sin t}$$

$$\text{I.F.} = e^{\frac{\tan^{-1} x^3}{\sqrt{1+x^6}} - \frac{x^3}{\sqrt{1+x^6}}}$$

Solution is

$$y\left(e^{\frac{\tan^{-1} x^3}{\sqrt{1+x^6}} - \frac{x^3}{\sqrt{1+x^6}}}\right) = \int 2x\, e^{\left(\frac{x^3 - \tan^{-1} x^3}{\sqrt{1+x^6}}\right)} \cdot e^{\frac{\tan^{-1} x^3 - x^3}{\sqrt{1+x^6}}} dx$$

$$y\left(e^{\frac{\tan^{-1} x^3 - x^3}{\sqrt{1+x^6}}}\right) = \int 2x\, dx = x^2 + c$$

above eq. is passing through (0, 0) then $c = 0$

$$y = x^2\, e^{\frac{x^3 - \tan^{-1} x^3}{\sqrt{1+x^6}}}$$

Put $x = 1$ then

$$y(1) = e^{\frac{1-\frac{\pi}{4}}{\sqrt{2}}} = e^{\frac{4-\pi}{4\sqrt{2}}}$$

$$\Rightarrow y(1) = \exp\left(\frac{4-\pi}{4\sqrt{2}}\right)$$

79. If the coefficient of x^{15} in the expansion of $\left(ax^3 + \frac{1}{bx^{1/3}}\right)^{15}$ is equal to the coefficient of x^{-15} in the expansion of $\left(ax^{1/3} - \frac{1}{bx^3}\right)^{15}$, where a and b are positive real numbers, then for each such ordered pair (a, b) :

(1) $ab = 3$ (2) $ab = 1$ (3) $a = b$ (4) $a = 3b$

Sol. **2**

$$\left(ax^3 + \frac{1}{bx^{1/3}}\right)^{15}$$

general term is $T_{r+1} = {}^{15}C_r\,(ax^3)^{15-r}\left(\frac{1}{bx^{1/3}}\right)^r$

$\Rightarrow T_{r+1} = {}^{15}C_r\,\frac{a^{15-r}}{b^r}\,x^{45-3r-\frac{r}{3}}$

For coefficient of $x^{15} \Rightarrow 45 - 3r - \frac{r}{3} = 15$

$$30 = \frac{10r}{3}$$

$$r = 9$$

Coefficient of x^{15} is $= {}^{15}C_9\,a^6\,b^{-9}$ … (1)

$\because$ general term of $\left(ax^{1/3} - \frac{1}{bx^3}\right)^{15}$ is

$T_{r+1} = {}^{15}C_r\,(ax^{1/3})^{15-r}\left(\frac{-1}{bx^3}\right)^r$

For coefficient of $x^{-15} \Rightarrow \frac{15-r}{3} - 3r = -15$

$\Rightarrow 15 - r - 9r = -45$

$\Rightarrow 60 = 10r$

$\boxed{r = 6}$

Coefficient of x^{-15} is $= {}^{15}C_6\,a^9\,b^{-b}$ … (2)

$\because$ both coefficient are equal then

${}^{15}C_9\,a^6\,b^{-9} = {}^{15}C_6\,a^9\,b^{-6}$

$\Rightarrow a^6\,b^{-9} = a^9\,b^{-6}$

$\Rightarrow a^3\,b^3 = 1$

$\Rightarrow \boxed{ab = 1}$

80. Suppose $f: \mathbb{R} \to (0, \infty)$ be a differentiable function such that $5f(x+y) = f(x) \cdot f(y), \forall x, y \in \mathbb{R}$. If $f(3) = 320$, then $\sum_{n=0}^{5} f(n)$ is equal to :

(1) 6875 (2) 6525 (3) 6825 (4) 6575

Sol. **3**

$f : \mathbb{R} \to (0, \infty)$

$5\,f(x+y) = f(x) \cdot f(y)$

Put x = 3, y = 0 then

$5\,f(3) = f(3)\,f(0)$

$\Rightarrow \boxed{f(0) = 5}$

Put x = 1, y = 1 then $5\,f(2) = f^2(1)$

Put x = 1, y = 2 then $5\,f(3) = f(1)\,f(2)$

$$5 \times 320 = \frac{f^3(1)}{5} = f(1) = 20$$

$$\Rightarrow f(2) = 80$$

Put x = 2, y = 2 then $5\,f(4) = f(2)\,f(2)$

$$f(4) = \frac{80 \times 80}{5} = 1280$$

Put x = 2, y = 3 then $5\,f(5) = f(2) \cdot f(2)$

$$f(5) = \frac{80 \times 320}{5} = 5120$$

$$\sum_{n=0}^{5} F(n) = f(0) + f(1) + \ldots. + f(5)$$

$$= 5 + 20 + 80 + 320 + 1280 + 5120$$

$$= 5(1 + 2^2 + 2^4 + 2^6 + 2^8 + 2^{10})$$

$$= 6825$$

Section B

81. Let $z = 1 + i$ and $z_1 = \frac{1+i\bar{z}}{\bar{z}(1-z)+\frac{1}{z}}$. Then $\frac{12}{\pi}\arg(z_1)$ is equal to

Sol. **9**

$z = 1 + i$, $\bar{z} = 1 - i$, $i\,\bar{z} = 1 + i$

$$z_1 = \frac{1 + i\bar{z}}{\bar{z}(1 - z) + \frac{1}{z}}$$

$$z_1 = \frac{i + z}{\bar{z} - z\bar{z}) + \frac{1}{z}}$$

$$z_1 = \frac{i + 2}{1 - i - 2 + \frac{1 - i}{2}}$$

$$z_1 = \frac{i + 2}{-\frac{1}{2} - \frac{3i}{2}}$$

$z_1 = \frac{-2(i+2)}{(1+3i)} \times \frac{(1-3i)}{(1-3i)}$

$z_1 = \frac{-2(5-5i)}{10}$

$z_1 = -1 + i$

$\arg.(z_1) = \pi - \tan^{-1}\left(\frac{1}{1}\right) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

$\therefore \arg(z_1) = \frac{12}{\pi} \times \frac{3\pi}{4} = 9$

82. If $\lambda_1 < \lambda_2$ are two values of λ such that the angle between the planes $P_1: \vec{r} \cdot (3\hat{i} - 5\hat{j} + \hat{k}) = 7$ and $P_2: \vec{r} \cdot (\lambda\hat{i} + \hat{j} - 3\hat{k}) = 9$ is $\sin^{-1}\left(\frac{2\sqrt{6}}{5}\right)$, then the square of the length of perpendicular from the point $(38\lambda_1, 10\lambda_2, 2)$ to the plane P_1 is

Sol. **315s**

Plane $P_1 : \vec{r}.(3\hat{i} - 5\hat{j} + \hat{k}) = 7$

$P_2 : \vec{r}.\left(\lambda\hat{i} + \hat{j} - 3k\right) = 9$

angle between plane is same as angle between their normal.

angle between normal θ then

$\text{Cos}\,\theta = \frac{\langle 3, -5, 1\rangle \cdot \langle \lambda, 1, -3\rangle}{\sqrt{9+25+1}\sqrt{\lambda^2+1+9}}$

$\text{Cos}\,\theta = \frac{3\lambda - 5 - 3}{\sqrt{35}\sqrt{\lambda^2+10}}$ … (1)

$\because \quad \theta = \sin^{-1}\left(\frac{2\sqrt{6}}{5}\right)$ then

$\sin\theta = \frac{2\sqrt{6}}{5}$

$\cos\theta = \frac{1}{5}$

from equation (1)

$\frac{3\lambda - 8}{\sqrt{35}\sqrt{\lambda^2+10}} = \frac{1}{5}$

$\Rightarrow \frac{(3\lambda-8)^2}{35(\lambda^2+10)} = \frac{1}{25}$

$\Rightarrow 5(3\lambda - 8)^2 = 7(\lambda^2 + 10)$

$\Rightarrow 5(9\lambda^2 - 48\lambda + 64) = 7\lambda^2 + 70$

$\Rightarrow 38\lambda^2 - 240\lambda + 250 = 0$

$\Rightarrow 19\lambda^2 - 120\lambda + 125 = 0$

$\Rightarrow \lambda = 5, \frac{25}{19}$

$\lambda_1 = \frac{25}{19}, \lambda_2 = 5$

Point $(38\lambda_1, 10\lambda_2, 2) \equiv (50, 50, 2)$

distance of (50, 50, 2) from plane P_1 is

$$d = \left|\frac{3 \times 50 - 5 \times 50 + 2 - 7}{\sqrt{9 + 25 + 1}}\right|$$

$$d = \left|\frac{150 - 250 + 2 - 7}{\sqrt{35}}\right|$$

$$d = \left|\frac{105}{\sqrt{35}}\right|$$

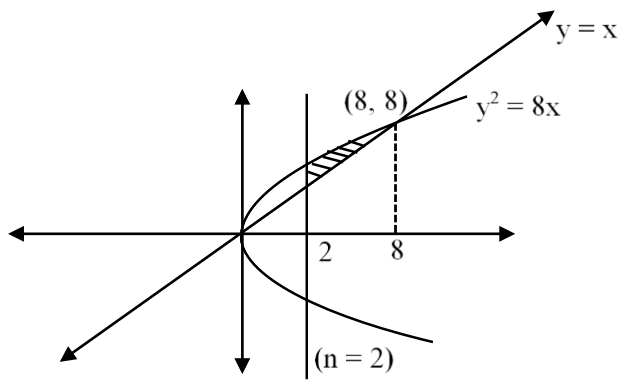
$$d = 3\sqrt{35}$$

$$\boxed{d^2 = 315}$$

83. Let α be the area of the larger region bounded by the curve $y^2 = 8x$ and the lines $y = x$ and $x = 2$, which lies in the first quadrant. Then the value of 3α is equal to

Sol. **22**

$$\text{area } (\alpha) = \int_2^8 (2\sqrt{2}\sqrt{x} - x)dx$$



$$= \left[2\sqrt{2}\cdot\frac{2}{3}x^{3/2} - \frac{x^2}{2}\right]_2^8$$

$$= \frac{4\sqrt{2}}{3}[8 \times 2\sqrt{2} - 2\sqrt{2}] - 30$$

$$= \frac{28 \times 4}{3} - 30$$

$$= \frac{112}{3} - 30$$

$$\alpha = \frac{22}{3}$$

$$\boxed{3\alpha = 22}$$

84. Let $\sum_{n=0}^{\infty} \frac{n^3((2n)!)+(2n-1)(n!)}{(n!)((2n)!)} = ae + \frac{b}{e} + c$, where $a, b, c \in \mathbb{Z}$ and $e = \sum_{n=0}^{\infty} \frac{1}{n!}$ Then $a^2 - b + c$ is equal to

Sol. **26**

Let $\sum_{n=0}^{\infty} \frac{n^3((2n)!) + (2n-1)n!}{(n!)((2n)!)}$

$= \sum_{n=0}^{\infty} \frac{n^3(2n)!}{n!(2n)!} + \frac{(2n-1)n!}{n!(2n)!}$

$= \quad S_1 \quad + \quad S_2$

Let $S_1 = \sum_{n=0}^{\infty} \frac{n^3(2n)!}{n!(2n)!} = \sum_{n=0}^{\infty} \frac{n^3}{n!} = \sum_{n=1}^{\infty} \frac{n^2}{(n-1)!}$

$= \sum_{n=1}^{\infty} \frac{n^2 - 1 + 1}{(n-1)!}$

$= \sum_{n=2}^{\infty} \frac{(n+1)}{(n-2)!} + \sum_{n=1}^{\infty} \frac{1}{(n-1)!}$

$= \sum_{n=2}^{\infty} \frac{(n-2)+3}{(n-2)!} + \sum_{n=1}^{\infty} \frac{1}{(n-1)!}$

$= \sum_{n=3}^{\infty} \frac{1}{(n-3)!} + 3\sum_{n=2}^{\infty} \frac{1}{(n-2)!} + \sum_{n=1}^{\infty} \frac{1}{(n-1)!}$

$S_1 = e + 3e + e = 5e$

$\because \quad S_2 = \sum_{n=0}^{\infty} \frac{(2n-1)n!}{n!(2n)!}$

$= \sum_{n=0}^{\infty} \frac{2n-1}{(2n)!}$

$= \sum_{n=1}^{\infty} \frac{1}{(2n-1)!} - \sum_{n=0}^{\infty} \frac{1}{(2n)!}$

$= \left(\frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \ldots\right) - \left(1 + \frac{1}{2!} + \frac{1}{4!} + \ldots\ldots\right)$

$= -1 + \frac{1}{1!} - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \frac{1}{5!} - \ldots$

$= -\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} + \ldots\right)$

$= -e^{-1}$

$S_1 + S_2 = 5e - \frac{1}{e} = ae + \frac{b}{e} + c$

Compare both side

a = 5, b = –1, c = 0

$a^2 - b + c = 25 + 1 + 0 = 26$

85. If the equation of the plane passing through the point (1,1,2) and perpendicular to the line $x - 3y + 2z - 1 = 0 = 4x - y + z$ is $Ax + By + Cz = 1$, then $140(C - B + A)$ is equal to

Sol. **15**

give line is $x – 3y + 2z – 1 = 0 = 4x – y + z$

Direction of line $\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & -3 & 2 \\ 4 & -1 & 1 \end{vmatrix} = \hat{i}(-1) - \hat{j}(-7) + k(11)$

$\Rightarrow \quad \vec{a} = <-1, 7, 11>$

$\because$ Line is $\perp^r$ to the plane then direction of line is parallel to normal of plane.

$\vec{n} = <-1, 7, 11>$

Equation of plane is

$-1(x-1) + 7(y-1) + 11(z-2) = 0$

$-x + 7y + 11z + 1 - 7 - 22 = 0$

$\Rightarrow \quad -x + 7y + 11z = 28$

$\Rightarrow \quad -\frac{1}{28}x + \frac{7}{28}y + \frac{11}{28}z = 1$

$A = -\frac{1}{28}, B = \frac{7}{28}, C = \frac{11}{28}$

$140(C - B + A) = 140\left(\frac{11}{28} - \frac{7}{28} - \frac{1}{28}\right)$

$= \frac{140 \times 3}{28} = 15$

86. Number of 4-digit numbers (the repeation of digits is allowed) which are made using the digits 1, 2, 3 and 5 , and are divisible by 15 , is equal to

Sol. **21**

			5

Last digit must be 5 and sum of digits is divisible by 3 for divisible by 15

Remaining 3 digits	Arrange
(1, 1, 2)	$\frac{3!}{2} = 3$
(1, 3, 3)	$\frac{3!}{2} = 3$
(1, 5, 1)	$\frac{3!}{2} = 3$
(2, 2, 3)	$\frac{3!}{2} = 3$
(2, 3, 5)	3! = 6
(3, 5, 5)	$\frac{3!}{2} = 3$

Total numbers = 21

87. Let $f^1(x) = \frac{3x+2}{2x+3}, x \in \mathbf{R} - \left\{\frac{-3}{2}\right\}$

For $n \geq 2$, define $f^n(x) = f^1$ of $f^{n-1}(x)$

If $f^5(x) = \frac{ax+b}{bx+a}, \gcd(a, b) = 1$, then $a + b$ is equal to

Sol. **3125**

$f^1(x) = \frac{3x+2}{2x+3}, x \in R - \left\{-\frac{3}{2}\right\}$

$f^2(x) = f^1 0\, f^1(x) = f^1\left(\frac{3x+2}{2x+3}\right)$

$$= \frac{3\left(\frac{3x+2)}{2x+3)}\right)+2}{2\left(\frac{3x+2}{2x+3}\right)+3}$$

$$= \frac{9x+6+4x+6}{6x+4+6x+9}$$

$$f^2(x) = \frac{13x+12}{12x+13}$$

$f^3(x) = f^1 \text{ o } f^2(x)$

$$= f^1\left(\frac{13x+12}{12x+13}\right)$$

$$= \frac{3\left(\frac{13x+12}{12x+13}\right)+2}{2\left(\frac{13x+12}{12x+13}\right)+3}$$

$$= \frac{39x+36+24x+26}{26x+24+36x+39}$$

$$f^3(x) = \frac{63x+62}{62x+63}$$

$$f^4(x) = f^1\left(\frac{63x+62}{62x+63}\right)$$

$$= \frac{3\left(\frac{63x+62}{62x+63}\right)+2}{2\left(\frac{63x}{62x}+\frac{62}{63}\right)+2}$$

$$f^4(x) = \frac{313x+312}{312x+313}$$

$$f^5(x) = f^1\left(\frac{313x+312}{312x+313}\right)$$

$$= \frac{3\left(\frac{313x+312}{312x+313}\right)+2}{2\left(\frac{313x+312}{312x+313}\right)+3}$$

$$f^5(x) = \frac{1563x+1562}{1562x+1563}$$

$$\because a = 1563, b = 1562$$

$$\boxed{a+b=3125}$$

88. The mean and variance of 7 observations are 8 and 16 respectively. If one observation 14 is omitted and a and b are respectively mean and variance of remaining 6 observation, then $a + 3b - 5$ is equal to

Sol. **37**

Mean of 7 observations = $\frac{\sum_{i=1}^{7} x_i}{7}$

$\Rightarrow \quad \sum_{i=1}^{7} x_i = 7 \times 8 = 56$

Variance = $\frac{\Sigma x_i^2}{n} - (\bar{x})^2$

$\Sigma x_i^2 = 7(16+64) = 560$

If 14 is removed then

Mean = $a = \frac{\sum_{i=1}^{7} x_i - 14}{6} \Rightarrow 6a = 56 - 14$

$\Rightarrow a = 7$

Variance = $b = \frac{\sum_{i=1}^{7} x_i^2 - (14)^2}{6} - 49$

$\Rightarrow \quad 6b = 560 - 196 - 294$

$\Rightarrow \quad 6b = 70$

$\Rightarrow \quad 3b = 35$

$\therefore \quad a + 3b - 5 = 7 + 35 - 5 = 37$

89. Let $S = \{1,2,3,4,5,6\}$. Then the number of one-one functions $f: S \to P(S)$, where $P(S)$ denote the power set of S, such that $f(n) \subset f(m)$ where $n < m$ is

Sol. **3240**

Case – I

f(6) = S i.e. 1 option

f(5) = any 5 element subset A of S i.e. ${}^6C_5 = 6$ options

f(4) = any 4 element subset B of A i.e. ${}^5C_4 = 5$ options

f(3) = any 3 element subset C of B i.e. ${}^4C_3 = 4$ options

f(2) = any 2 element subset D of C i.e. ${}^3C_2 = 3$ options

f(1) = any 1 element subset E of D or empty subset i.e. 3 options

Total function = 6×5×4×3×2×3=1080

Case – II

f(6) = S

f(5) = any 4 element subset A of S i.e. ${}^6C_4 = 15$ options

f(4) = any 3 element subset B of A i.e. ${}^4C_3 = 4$ options

f(3) = any 2 element subset C of B i.e. ${}^3C_2 = 2$ options

f(2) = any 1 element subset D of C i.e. ${}^2C_1 = 2$ options

f(1) = empty subset i.e. 1 options

Total function = 15×4×3×2×1=360

Case – III

f(6) = S

f(5) = any 5 element subset A of S i.e. ${}^6C_5 = 6$ options

f(4) = any 3 element subset B of A i.e. ${}^5C_3 = 10$ options

f(3) = any 2 element subset C of B i.e. ${}^3C_2 = 3$ options

f(2) = any 1 element subset D of C i.e. ${}^2C_1 = 2$ options

f(1) = empty subset i.e. 1 options

Total function = 6×10×3×2×1=360

Case – IV

f(6) = S

f(5) = any 5 element subset A of S i.e. ${}^6C_5 = 6$ options

f(4) = any 4 element subset B of A i.e. ${}^5C_4 = 5$ options

f(3) = any 2 element subset C of B i.e. ${}^4C_2 = 6$ options

f(2) = any 1 element subset D of C i.e. ${}^2C_1 = 2$ options

f(1) = empty subset i.e. 1 options

Total function = 6×5×6×2×1=360

Case – V

f(6) = S

f(5) = any 5 element subset A of S i.e. $^6C_5 = 6$ options

f(4) = any 4 element subset B of A i.e. $^5C_4 = 5$ options

f(3) = any 3 element subset C of B i.e. $^4C_3 = 4$ options

f(2) = any 1 element subset D of C i.e. $^3C_1 = 3$ options

f(1) = empty subset i.e. 1 options

Total function = 6×5×4×3×1=360

Case – VI

f(6) = any 5 element subset A of S i.e. $^6C_5 = 6$ options

f(5) = any 4 element subset B of A i.e. $^5C_4 = 5$ options

f(4) = any 3 element subset C of B i.e. $^4C_3 = 4$ options

f(3) = any 2 element subset D of C i.e. $^3C_2 = 3$ options

f(2) = any 1 element subset E of D i.e. $^2C_1 = 2$ options

f(1) = empty subset i.e. 1 options

Total function = 6×5×4×3×2×1=720

Total number of such functions = 1080 + (4× 360) +720 = **3240**

90. $\lim_{x\to 0} \frac{48}{x^4} \int_0^x \frac{t^3}{t^6+1}$ dt is equal to

Sol. **12**

$\lim_{x\to 0} \frac{48}{x^4} \int_0^x \frac{t^3}{t^6+1} = \left(\frac{0}{0}\right)$ form

Using L Hopital Rule

$$= \lim_{x\to 0} \frac{48 \times \frac{x^3}{x^6+1}}{4x^3}$$

$$= \lim_{x\to 0} \frac{48}{4(x^6+1)}$$

$$= \frac{48}{4}$$

= 12