JEE–MAIN EXAMINATION – JANUARY, 2023	
(Held On Thursday 31st January, 2023)	TIME : 9 : 00 AM to 12 : 00 NOON

Physics

SECTION - A

1. The maximum potential energy of a block executing simple harmonic motion is 25 J. A is amplitude of oscillation. At $A/2$, the kinetic energy of the block is :

(1) 18.75 J (2) 9.75 J (3) 37.5 J (4) 12.5 J

Sol. **(1)**

Total Energy in SHM, $E=\frac{1}{2}m\omega^2A^2=25J$

$$\text{at}\frac{A}{2},U=PE=\frac{1}{2}m\omega^2x^2$$

$$U=\frac{1}{2}m\omega^2\left(\frac{A}{2}\right)^2$$

$$k+U=E$$

$$k=\frac{1}{2}m\omega^2A^2\left(1-\frac{1}{4}\right)$$

$$k=25\times\frac{3}{4}=18.75J$$

2. The drift velocity of electrons for a conductor connected in an electrical circuit is V_d. The conductor in now replaced by another conductor with same material and same length but double the area of cross section. The applied voltage remains same. The new drift velocity of electrons will be

(1) V_d (2) $\frac{V_d}{4}$ (3) 2 V_d (4) $\frac{V_d}{2}$

Sol. **(1)**

$$V=IR=I\left(\frac{\rho l}{A}\right)$$

$$A\rightarrow 2A$$

$$I\rightarrow 2I$$

$$I=AneV_d$$

$$V_d\propto\frac{I}{A}$$

3. The initial speed of a projectile fired from ground is u. At the highest point during its motion, the speed of projectile is $\frac{\sqrt{3}}{2}u$. The time of flight of the projectile is :

(1) $\frac{2u}{g}$ (2) $\frac{u}{2g}$ (3) $\frac{\sqrt{3}u}{g}$ (4) $\frac{u}{g}$

Sol. **(4)**

At highest point -

$$u\cos\theta=\frac{\sqrt{3}u}{2}$$

$$\theta=30^\circ$$

$$T=\frac{2u\sin\theta}{g}=\frac{u}{g}$$

4. The correct relation between $\gamma = \frac{c_p}{c_v}$ and temperature T is :

(1) $\gamma \alpha T^0$ (2) $\gamma \alpha \mathrm{T}$ (3) $\gamma \alpha \frac{1}{\sqrt{\mathrm{T}}}$ (4) $\gamma \alpha \frac{1}{\mathrm{T}}$

Sol. **(1)**

$\gamma = \frac{C_p}{C_v}$, Independent on T

5. The effect of increase in temperature on the number of electrons in conduction band (n_e) and resistance of a semiconductor will be as:

(1) Both n_e and resistance increase (2) Both n_e and resistance decrease

(3) n_e decreases, resistance increases (4) n_e increases, resistance decreases

Sol. **(4)**

In semi conductors,

$T\uparrow, n_e$ in Conduction Band increases

$T\uparrow, R\downarrow$

6. The amplitude of $15\sin(1000\pi t)$ is modulated by $10\sin(4\pi t)$ signal. The amplitude modulated signal contains frequency (ies) of

A. 500 Hz B. 2 Hz C. 250 Hz D. 498 Hz E. 502 Hz

Choose the correct answer from the options given below:

(1) A Only (2) B Only (3) A and B Only (4) A, D and E Only

Sol. **(4)**

$$f_c = \frac{1000\pi}{2\pi} = 500\text{Hz}$$

$$f_m = \frac{4\pi}{2\pi} = 2\text{Hz}$$

Upper side Band, $\text{USB} = f_c + f_m$

$\text{USB} = 502\text{HZ}$

Lower side Band, $\begin{aligned} LSB &= f_c - f_m \\ LSB &= 498\text{Hz} \end{aligned}$

7. Two polaroide A and B are placed in such a way that the pass-axis of polaroids are perpendicular to each other. Now, another polaroid C is placed between A and B bisecting angle between them. If intensity of unpolarized light is I_0 then intensity of transmitted light after passing through polaroid B will be:

(1) $\frac{I_0}{4}$ (2) $\frac{I_0}{2}$ (3) Zero (4) $\frac{I_0}{8}$

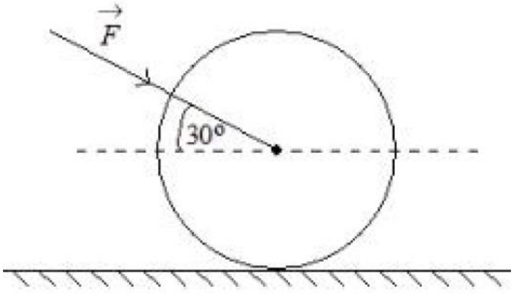
Sol. **(4)**

After A, $I = \frac{I_\circ}{2}$

After C, $I = \frac{I_0}{2}\cos^2 45^\circ = \frac{I_0}{4}$

After B, $I = \frac{I_0}{4}\cos^2 45^\circ = \frac{I_0}{8}$

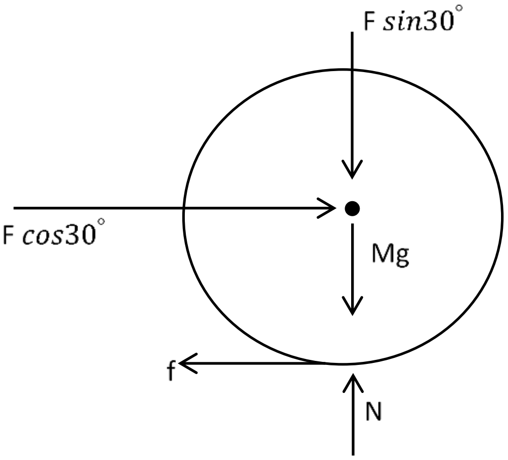
8. As shown in figure, a 70 kg garden roller is pushed with a force of $\vec{F} = 200$ N at an angle of 30° with horizontal. The normal reaction on the roller is
(Given $g = 10 \text{ m s}^{-2}$)



(1) $800\sqrt{2}$ N (2) $200\sqrt{3}$ N (3) 600 N (4) 800 N

Sol. **(4)**

FBD of Sphere ⇒



N= Mg + F$sin30^{\circ}$

N= 700 +200 $sin30^{\circ}$

N= 800 N

9. If 1000 droplets of water of surface tension 0.07 N/m, having same radius 1 mm each, combine to from a single drop. In the process the released surface energy is-
(Take $\pi = \frac{22}{7}$)

(1) 8.8×10^{-5} J (2) 7.92×10^{-4} J (3) 7.92×10^{-6} J (4) 9.68×10^{-4} J

Sol. **(2)**

$V_1 = V_2$

$1000 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

$R = 10\text{r}$

$E = U_1 - U_2$

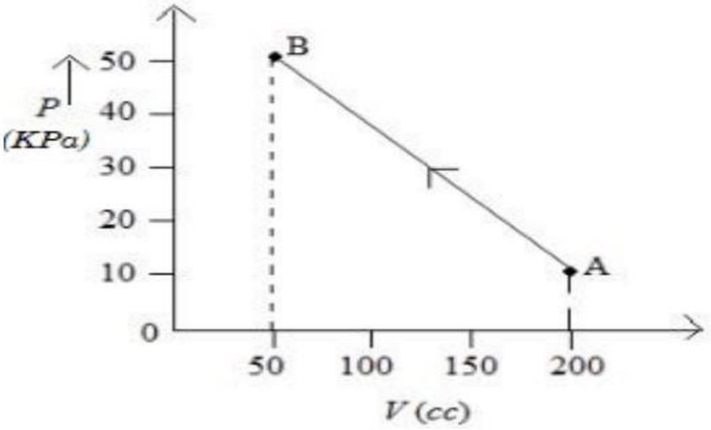
$= 1000\left(T \times 4\pi r^2\right) - T \times 4\pi R^2$

$E = 4\pi T\left(1000 \times r^2 - 100r^2\right)$

$E = 4 \times \frac{22}{7} \times 0.07 \times 900 \times 10^{-6}$

$E = 7.92 \times 10^{-4}$ J

10. The pressure of a gas changes linearly with volume from *A* to *B* as shown in figure. If no heat is supplied to or extracted from the gas then change in the internal energy of the gas will be



(1) −4.5 J (2) zero (3) 4.5 J (4) 6 J

Sol. **C**

W = Area of *PV* Graph

$W = -\frac{1}{2} \times [50 + 10] \times 10^3 \times 150 \times 10^{-6}$

$W = -4.5\text{J}$

$Q = \Delta U + W$

$0 = \Delta U - 4.5$

$= \Delta U = 4.5\text{J}$

11. Given below are two statements: One is labelled as Assertion **A** and the other is labelled as Reason **R**

Assertion A: The beam of electrons show wave nature and exhibit interference and diffraction.

Reason R: Davisson Germer Experimentally verified the wave nature of electrons.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both A and R are correct and R is the correct explanation of A
(2) A is not correct but R is correct
(3) A is correct but R is not correct
(4) Both A and R are correct but R is Not the correct explanation of A

Sol. **(1)**

Theoritical

12. A free neutron decays into a proton but a free proton does not decay into neutron. This is because

(1) proton is a charged particle
(2) neutron is an uncharged particle
(3) neutron is a composite particle made of a proton and an electron
(4) neutron has larger rest mass than proton

Sol. **(4)**

Rest mass of neutron is greater than proton.

13. Spherical insulating ball and a spherical metallic ball of same size and mass are dropped from the same height. Choose the correct statement out of the following Assume negligible air friction}

(1) Insulating ball will reach the earth's surface earlier than the metal ball

(2) Metal ball will reach the earth's surface earlier than the insulating ball

(3) Both will reach the earth's surface simultaneously.

(4) Time taken by them to reach the earth's surface will be independent of the properties of their materials

Sol. **(1)**

In Conductor, A portion of the Gravitational Potential Energy goes into generating eddy current.

14. If R, X_L, and X_C represent resistance, inductive reactance and capacitive reactance. Then which of the following is dimensionless :

(1) $\frac{R}{X_L X_C}$ (2) $\frac{R}{\sqrt{X_L X_C}}$ (3) $R\frac{X_L}{X_C}$ (4) $RX_L\, X_C$

Sol. **(2)**

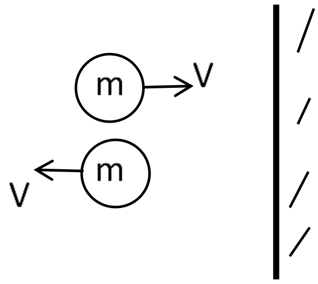
R, X_L, X_C have same unit i.e. ohm

$$\frac{R}{\sqrt{X_L X_C}} \rightarrow \frac{\text{ohm}}{\sqrt{\text{ohm}^2}} \rightarrow \text{Dimensionless}$$

15. 100 balls each of mass m moving with speed v simultaneously strike a wall normally and reflected back with same speed, in time t sec. The total force exerted by the balls on the wall is

(1) $\frac{100mv}{t}$ (2) $200mvt$ (3) $\frac{mv}{100t}$ (4) $\frac{200mv}{t}$

Sol. **(4)**



Change in momentum,

$|\overrightarrow{\Delta p}| = 2mV$

Average force,

$$F_{\text{avg}} = N\frac{|\overrightarrow{\Delta p}|}{t}$$

$$F_{\text{avg}} = 100\left(\frac{2mV}{t}\right)$$

$$F_{\text{avg}} = \frac{200mV}{t}$$

16. If a source of electromagnetic radiation having power 15 kW produces 10^{16} photons per second, the radiation belongs to a part of spectrum is.

(Take Planck constant $h = 6 \times 10^{-34}$Js)

(1) Micro waves (2) Ultraviolet rays (3) Gamma rays (4) Radio waves

Sol. **(3)**

$$P = \frac{N}{t}\left(\frac{hc}{\lambda}\right)$$

$$15 \times 10^{3} = 10^{16} \times \frac{6 \times 10^{-34} \times 3 \times 10^{8}}{\lambda}$$

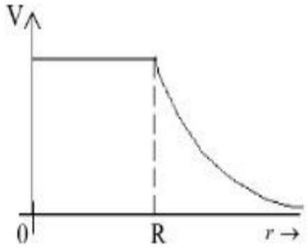
$$\lambda = 1.2 \times 10^{-13} m$$

$$\lambda = 0.0012 A^{0}$$

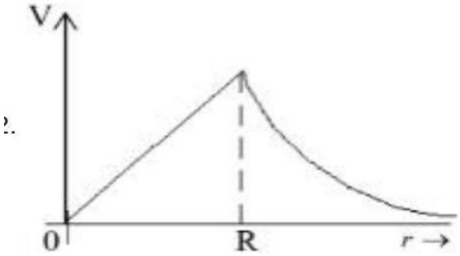
Corresponds to Gamma rays

17. Which of the following correctly represents the variation of electric potential (V) of a charged spherical conductor of radius (R) with radial distance (r) from the center?

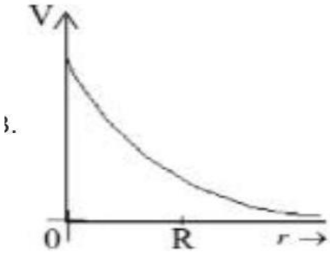
(1)



(2)



(3)



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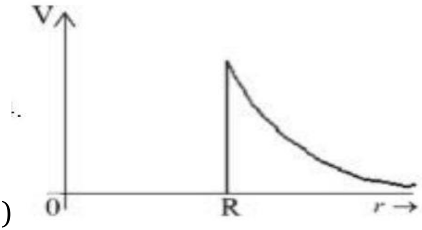
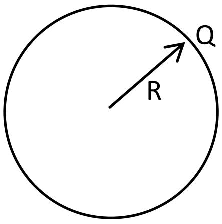
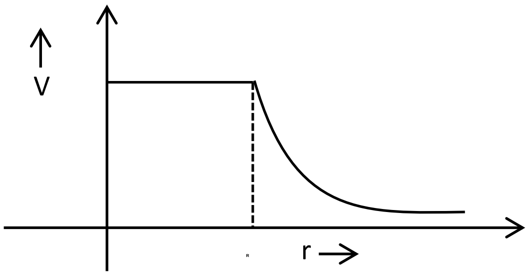


Sol. **(1)**



$$V_{in} = \frac{kQ}{R} \rightarrow \text{Constant}$$

$$V_{out} = \frac{kQ}{r} \propto \frac{1}{r}$$



18. A bar magnet with a magnetic moment $5.0\mathrm{Am}^2$ is placed in parallel position relative to a magnetic field of 0.4 T. The amount of required work done in turning the magnet from parallel to antiparallel position relative to the field direction is ________.

(1) 1 J (2) 4 J (3) 2 J (4) zero

Sol. **(2)**

$$W = MB(\cos\theta_1 - \cos\theta_2)$$

$$W = MB(\cos 0^\circ - \cos 180^\circ)$$

$$W = 2MB$$

$$W = 2 \times 5 \times 0.4$$

$$W = 4\,\mathrm{J}$$

19. At a certain depth "d " below surface of earth, value of acceleration due to gravity becomes four times that of its value at a height $3R$ above earth surface. Where R is Radius of earth (Take $R = 6400$ km). The depth d is equal to

(1) 4800 km (2) 2560 km (3) 640 km (4) 5260 km

Sol. **(A)**

Given

$$g\left(1 - \frac{d}{R}\right) = 4\frac{g}{\left(1 + \frac{h}{R}\right)^2}$$

$$1 - \frac{d}{R} = \frac{4}{(1+3)^2} = \frac{1}{4}$$

$$\frac{d}{R} = \frac{3}{4}$$

$$d = \frac{3R}{4} = \frac{3}{4} \times 6400$$

$$d = 4800\mathrm{km}$$

20. A rod with circular cross-section area $2\ \mathrm{cm}^2$ and length 40 cm is wound uniformly with 400 turns of an insulated wire. If a current of 0.4 A flows in the wire windings, the total magnetic flux produced inside windings is $4\pi \times 10^{-6}$ Wb. The relative permeability of the rod is

(Given : Permeability of vacuum $\mu_0 = 4\pi \times 10^{-7}\mathrm{NA}^{-2}$)

(1) $\frac{5}{16}$ (2) 12.5 (3) 125 (4) $\frac{32}{5}$

Sol. **(1)**

NTA Ans. (3)

Magnetic field in the Solenoid,

$$\mathrm{B} = \mu_0 \mu_r \mathrm{nI}$$

Magnetic flux, $\phi = \mathrm{N}(\mathrm{BA})$

$$\phi = N(\mu_0 \mu_r nIA)$$

$$4\pi \times 10^{-6} = 400\left(4\pi \times 10^{-7} \mu_y \times \frac{400}{0.4} \times 0.4 \times 2 \times 10^{-4}\right)$$

$$\frac{1}{40} = \mu_r \times 8 \times 10^{-2}$$

$$\mu_r = \frac{100}{320} = \frac{5}{16}$$

SECTION - B

21. In a medium the speed of light wave decreases to 0.2 times to its speed in free space The ratio of relative permittivity to the refractive index of the medium is $x:1$. The value of x is (Given speed of light in free space $= 3 \times 10^8$ m s^{-1} and for the given medium $\mu_r = 1$)

Sol. **(5)**

$$V = \frac{c}{n}$$

$n \rightarrow$ refractive index

$$n = \frac{c}{0.2c} = 5$$

$$n = \sqrt{\mu_r \varepsilon_r}$$

$$\varepsilon_r = n^2 = 25$$

$$\frac{\varepsilon_r}{n} = \frac{25}{5} = \frac{5}{1}$$

22. A solid sphere of mass 1 kg rolls without slipping on a plane surface. Its kinetic energy is 7×10^{-3} J. The speed of the centre of mass of the sphere is __________ cms^{-1}

Sol. **(10)**

On Rolling,

$$KE = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2$$

$$KE = \frac{1}{2}MV^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v}{R}\right)^2$$

$$KE = \frac{7}{10}MV^2 = 7 \times 10^{-3}$$

$$V^2 = 10^{-2}$$

$$V = 10^{-1}\text{m/s}$$

$$V = 10\text{cm/s}$$

23. A lift of mass $M = 500$ kg is descending with speed of 2 ms^{-1}. Its supporting cable begins to slip thus allowing it to fall with a constant acceleration of 2 ms^{-2}. The kinetic energy of the lift at the end of fall through to a distance of 6 m will be ________ kJ.

Sol. **(7)**

Acceleration is constant,

$$\text{v}^2 = u^2 + 2as$$

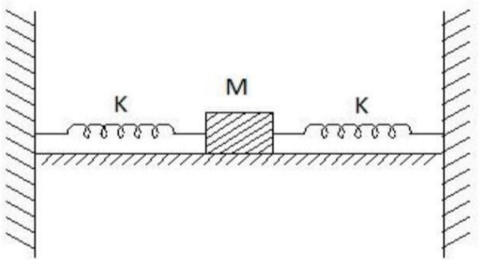
$$\text{v}^2 = 2^2 + 2(2)(6)$$

$$\text{v}^2 = 28$$

$$\frac{1}{2}\text{Mv}^2 = \frac{1}{2} \times 500 \times 28$$

$$\text{KE} = 7\text{kJ}$$

24. In the figure given below, a block of mass $M = 490$ g placed on a frictionless table is connected with two springs having same spring constant ($\mathrm{K} = 2\ \mathrm{N\ m}^{-1}$). If the block is horizontally displaced through ' X ' m then the number of complete oscillations it will make in 14π seconds will be ________.



Sol. **(20)**

$$T = 2\pi\sqrt{\frac{m}{k_{eq}}}$$

$$T = 2\pi\sqrt{\frac{m}{2k}}$$

$$T = 2\pi\sqrt{\frac{0.49}{2\times 2}}$$

$$T = 2\pi\times\frac{0.7}{2} = 0.7\pi$$

in $14\pi\sec, \frac{14\pi}{0.7\pi} = 20$

25. An inductor of 0.5mH, a capacitor of 20μF and resistance of 20Ω are connected in series with a 220 V ac source. If the current is in phase with the emf, the amplitude of current of the circuit is $\sqrt{x}$ A. The value of x is-

Sol. **(242)**

Current is in phase with EMF. Hence, Circuit is at Resonance.

$$I_{rms} = \frac{V_{rms}}{R} = \frac{220}{20}$$

$$I_{rms} = 11\mathrm{A}$$

$$I_0 = \sqrt{2}\, I_{rms} = \sqrt{242}\mathrm{A}$$

26. The speed of a swimmer is $4\ \mathrm{km\ h}^{-1}$ in still water. If the swimmer makes his strokes normal to the flow of river of width 1 km, he reaches a point 750 m down the stream on the opposite bank. The speed of the river water is ________ kmh^{-1}.

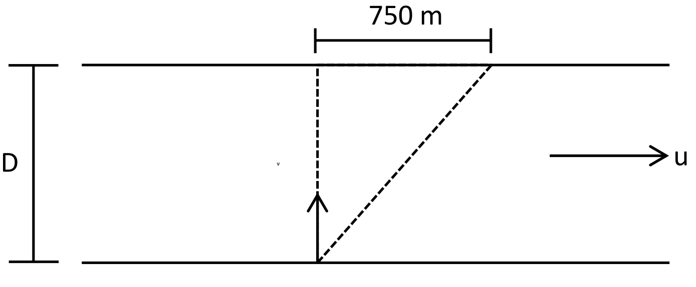
Sol. **(3)**

$$T = \frac{D}{V} = \frac{1}{4}hr$$

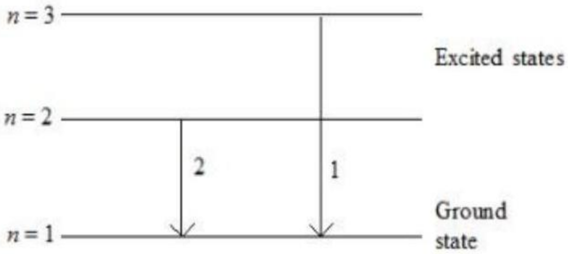
$$\text{Drift} = uT$$

$$\frac{750}{1000}\text{kM} = u\times\frac{1}{4}hr$$

$$u = 3\text{km/hr}$$



27. For hydrogen atom, λ_1 and λ_2 are the wavelengths corresponding to the transitions 1 and 2 respectively as shown in figure. The ratio of λ_1 and λ_2 is $\frac{x}{32}$. The value of x is _________.



Sol. **(27)**

$$\frac{1}{\lambda_1}=R\left(\frac{1}{1^2}-\frac{1}{3^2}\right)$$

$$\lambda_1=\frac{9}{8R}$$

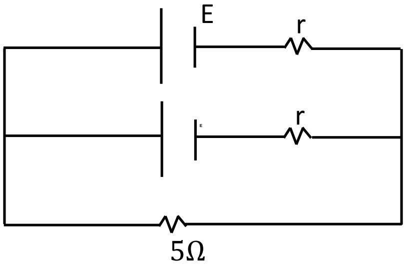
$$\frac{1}{\lambda_2}=R\left(\frac{1}{1^2}-\frac{1}{2^2}\right)$$

$$\lambda_2=\frac{4}{3R}$$

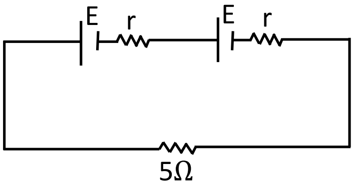
$$\frac{\lambda_1}{\lambda_2}=\frac{27}{32}$$

28. Two identical cells, when connected either in parallel or in series gives same current in an external resistance 5Ω. The internal resistance of each cell will be _______ Ω.

Sol. **(5)**



(I)



(II)

$$r_{eq}=\frac{r}{2}\ ,r_{eq}=2r$$

$$E_{eq}=\frac{r}{2}\left(\frac{E}{r}+\frac{E}{r}\right)=E\ ,\ E_{eq}=2E$$

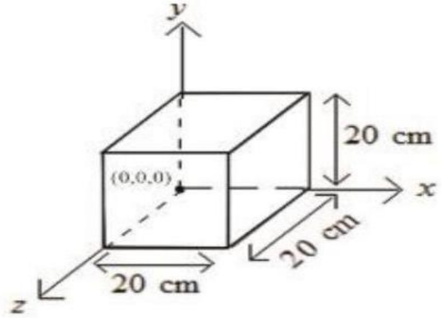
$$I_1=\frac{E}{5+\frac{r}{2}},\ I_2=\frac{2E}{2r+5}$$

$$I_1=I_2$$

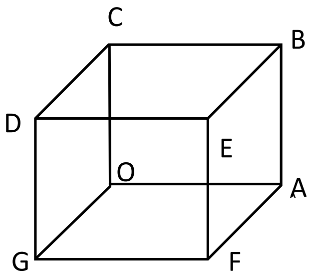
$$2r+5\ =2\left(5+\frac{r}{2}\right)$$

$$r\ =5\Omega$$

29. Expression for an electric field is given by $\vec{E} = 4000x^2\hat{\imath}\frac{V}{m}$. The electric flux through the cube of side 20 cm when placed in electric field (as shown in the figure) is ______ V cm



Sol. **(640)**



$\vec{E} \perp \vec{A}, \phi_{Top} = \phi_{Bottom} = \phi_{front} = \phi_{Back} = 0$

for $OCDG, x = 0, E = 0, \phi = 0$

for $ABEF, x = 0.2\text{m}$

$E = 4000 \times (0.2)^2$

$E = 160\text{V/m}$

$\phi = E\left(a^2\right) = 160\text{V/m} \times (0.2)^2\text{m}^2$

$\phi = 6.4\text{V} - \text{m}$

$\phi = 640\text{V} - \text{cm}$

30. A thin rod having a length of 1 m and area of cross-section 3×10^{-6} m^2 is suspended vertically from one end. The rod is cooled from 210°C to 160°C. After cooling, a mass M is attached at the lower end of the rod such that the length of rod again becomes 1 m. Young's modulus and coefficient of linear expansion of the rod are 2×10^{11} N m^{-2} and 2×10^{-5} K^{-1}, respectively. The value of M is ______ kg. (Take g = 10 m s^{-2})

Sol. **(60)**

$$Y = \frac{FL}{A\Delta L}$$

$$F = YA\left(\frac{\Delta L}{L}\right)$$

$$F = YA(\alpha\Delta T)$$

$$Mg = YA(\alpha\Delta T)$$

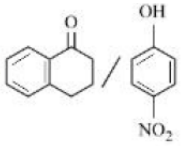
$$M \times 10 = 2 \times 10^{11} \times 3 \times 10^{-6} \times 2 \times 10^{-5} \times 50$$

$$M = 60\text{kg}$$

Chemistry

SECTION - A

31. Match items of column I and II

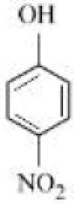
Column I (Mixture of compounds)	Column II (Separation Technique)
A. H_2O/CH_2Cl_2	i. Crystallization
B. 	ii. Differential solvent extraction
C. Kerosene /Naphthalene	iii. Column chromatography
D. $C_6H_{12}O_6/NaCl$	iv. Fractional Distillation

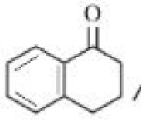
Correct match is

(1) A-(ii), B-(iii), C-(iv), D-(i)
(2) A-(i), B-(iii), C-(ii), D-(iv)
(3) A-(ii), B-(iv), C-(i), D-(iii)
(4) A-(iii), B-(iv), C-(ii), D-(i)

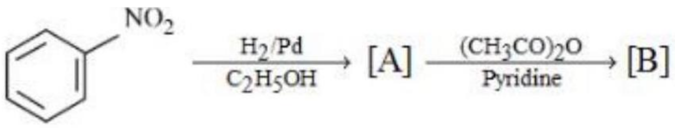
Sol. **1**

A-(ii),
Density of CH_2Cl_2 > Density of H2O
(Can separated by differential solvent extraction
B-(iii),



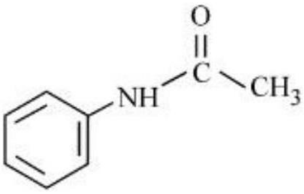
Having intermolecular H–Bond so can be separated from  through column chromatography

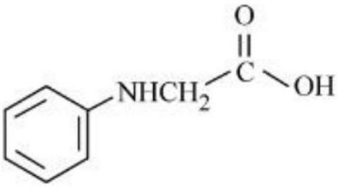
C-(iv),
Due to difference in B.P. of kerosene and Naphthalene, it can be separated by fractional distillation
D-(i)
NaCl → ionic compound
$C_6H_{12}O_6$ → Non ionic compound
so NaCl can by crystallized.



32.

Consider the above reaction and identify the product B. Options


(1)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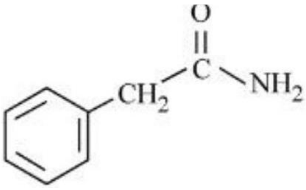
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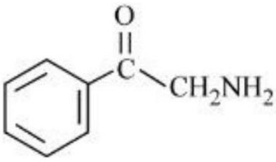
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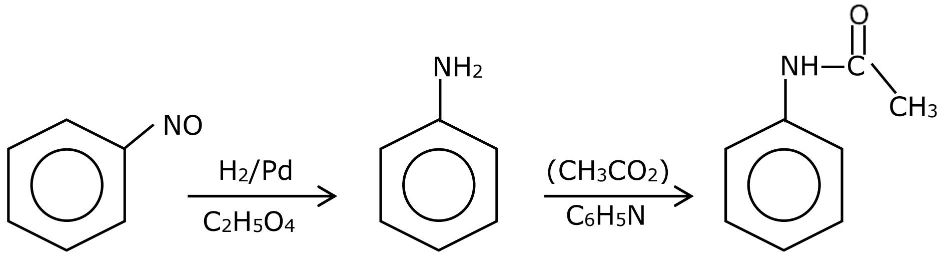
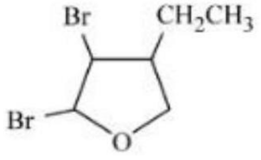
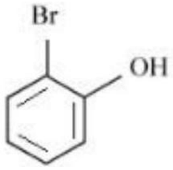
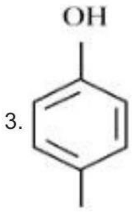
Sol. **1**

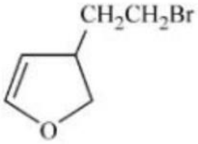


33. An organic compound 'A' with emperical formula C_6H_6O gives sooty flame on burning. Its reaction with bromine solution in low polarity solvent results in high yield of $B. B$ is

(1) 

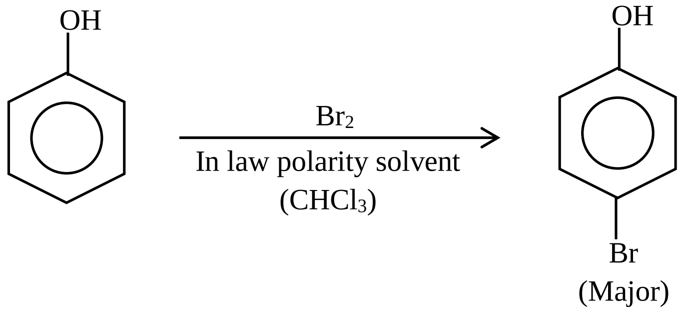
(2) 

(3) 

(4) 

Sol. **3**

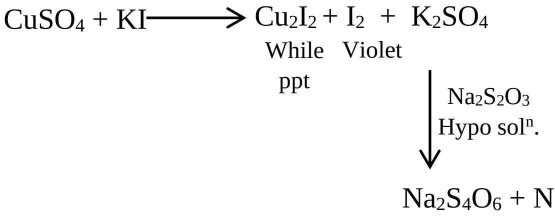
Phenol will give sooty flame while burning (aromatic compound)



34. When Cu^{2+} ion is treated with KI, a white precipitate, X appears in solution. The solution is titrated with sodium thiosulphate, the compound Y is formed. X and Y respectively are

(1) $X = CuI_2$ $Y = Na_2\ S_4O_6$
(2) $X = CuI_2$ $Y = Na_2\ S_2O_3$
(3) $X = Cu_2I_2$ $Y = Na_2\ S_4O_5$
(4) $X = Cu_2I_2$ $Y = Na_2\ S_4O_6$

Sol. **4**



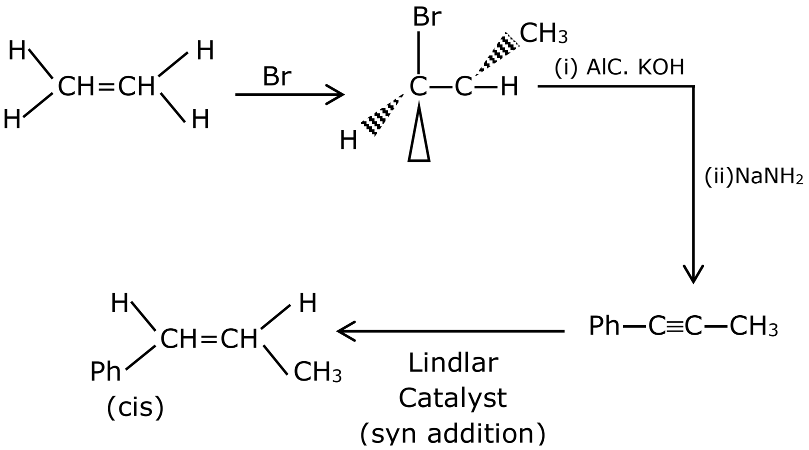
'M' Electrolysis & liquation is method of purification where as hydraulic washing, leading, froth flotation are method of can conbration.

35. Choose the correct set of reagents for the following conversion.

$\mathrm{trans(Ph-CH=CH-CH_3) \rightarrow cis(Ph-CH=CH-CH_3)}$

(1) $\mathrm{Br_2, aq \cdot KOH, NaNH_2, Na(LiqNH_3)}$
(2) $\mathrm{Br_2}$, alc $\cdot$ $\mathrm{KOH, NaNH_2, H_2}$ Lindlar Catalyst
(3) $\mathrm{Br_2, aq \cdot KOH, NaNH_2, H_2}$ Lindlar Catalyst
(4) $\mathrm{Br_2}$, alc $\cdot$ $\mathrm{KOH, NaNH_2, Na(LiqNH_3)}$

Sol. **2**

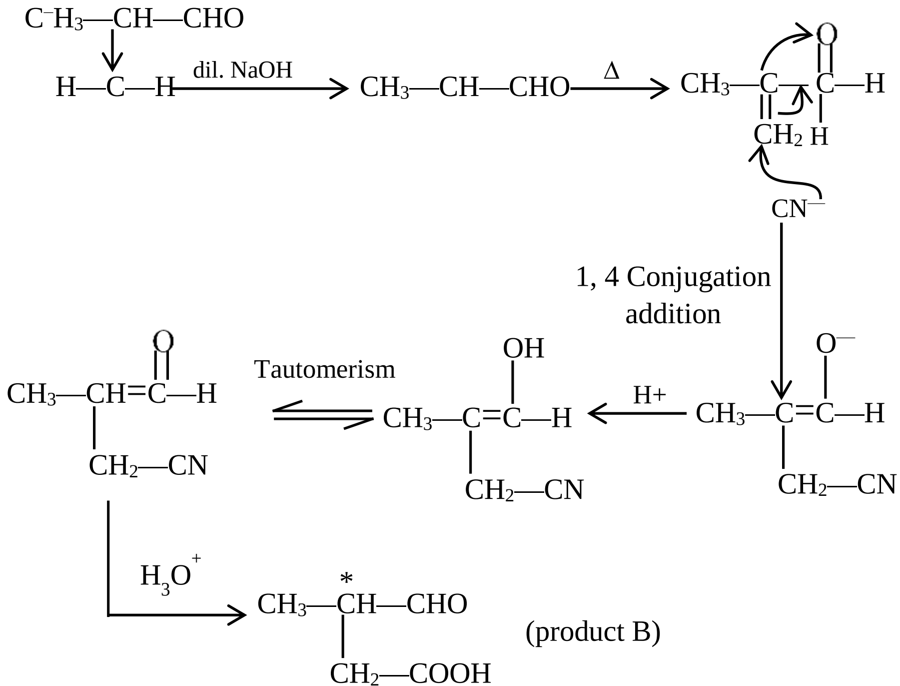


36. Consider the following reaction

$$\text{Propanal} + \text{Methanal} \xrightarrow[\text{(ii) }\Delta\text{ (iii) NaCN (iv) }H_3O^+]{\text{(i) dil.NaOH}} \text{Product B } (C_5H_8O_3)$$

The correct statement for product B is. It is
(1) optically active alcohol and is neutral
(2) racemic mixture and gives a gas with saturated $\mathrm{NaHCO_3}$ solution
(3) optically active and adds one mole of bromine
(4) racemic mixture and is neutral

Sol. **2**



Carboxylic acid will give CO_2 gas with $NaHCO_3$ solutions

37. The methods NOT involved in concentration of ore are

A. Liquation B. Leaching C. Electrolysis D. Hydraulic washing

E. Froth floatation

Choose the correct answer from the options given below :

(1) C, D and E only (2) B, D and C only (3) A and C only (4) B, D and E only

Sol. **3**

Methods involved in concentration of one are

(i) Hydraulic Washing

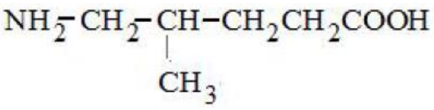
(ii) Froth Flotation

(iii) Magnetic Separation

(iv) Leaching

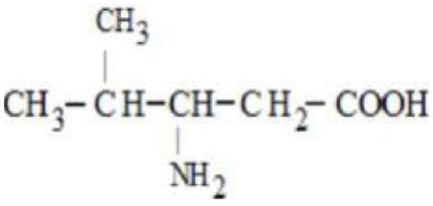
38. A protein 'X' with molecular weight of 70,000u, on hydrolysis gives amino acids. One of these amino acid is

(1)



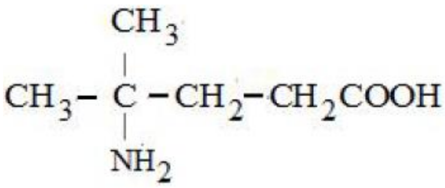
$NH_2-CH_2-\underset{\underset{CH_3}{|}}{CH}-CH_2CH_2COOH$

(2)



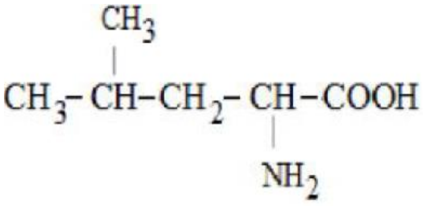
$CH_3-\overset{\overset{CH_3}{|}}{CH}-\underset{\underset{NH_2}{|}}{CH}-CH_2-COOH$

(3)



$CH_3-\underset{\underset{NH_2}{|}}{\overset{\overset{CH_3}{|}}{C}}-CH_2-CH_2COOH$

(4)



$CH_3-\overset{\overset{CH_3}{|}}{CH}-CH_2-\underset{\underset{NH_2}{|}}{CH}-COOH$

Sol. **4**

From protein, only $\propto$–Amino acid is possible so answer is (4).

39. $Nd^{2+} =$

(1) $4f^3$ (2) $4f^4 6\ s^2$ (3) $4f^4$ (4) $4f^2 6\ s^2$

Sol. **3**

Na = $4f^4\ 5d^0\ 6s^2$

Na^{+2} = $4f^4\ 5d^0\ 6s^0$

40. Match List I with List II

List I	List II
A. XeF_4	I. See-saw
B. SF_4	II. Square planar
C. NH_4^+	III. Bent T-shaped
D. BrF_3	IV. Tetrahedral

Choose the correct answer from the options given below :

(1) A-IV, B-III, C-II, D-I (2) A-IV, B-I, C-II, D-III

(3) A-II, B-I, C-III, D-IV (4) A-II, B-I, C-IV, D-III

Sol. **4**

XeF_4	Sq. planar
SF_4	see saw
NH_4^+	Tetrahedral
BrF_3	Bent 'T' shaped.

41. Identify X, Y and Z in the following reaction. (Equation not balanced)

$$ClO + NO_2 \rightarrow \underline{X} \xrightarrow{H_2O} \underline{Y} + \underline{Z}$$

(1) $X = ClONO_2, Y = HOCl, Z = HNO_3$

(2) $X = ClONO_2, Y = HOCl, Z = NO_2$

(3) $X = ClNO_2, Y = HCl, Z = HNO_3$

(4) $X = ClNO_3, Y = Cl_2, Z = NO_2$

Sol. **1**

$$Cl\overset{\circ}{O} + NO_2 \longrightarrow \underset{(x)}{ClO.NO_2} \xrightarrow{H_2O} \underset{(y)}{HOCl} + \underset{(z)}{HNO_3}$$

42. The correct increasing order of the ionic radii is

(1) $S^{2-} < Cl^- < Ca^{2+} < K^+$

(2) $K^+ < S^{2-} < Ca^{2+} < Cl^-$

(3) $Ca^{2+} < K^+ < Cl^- < S^{2-}$

(4) $Cl^- < Ca^{2+} < K^+ < S^{2-}$

Sol. **3**

For isoelectronic species size $\propto \frac{1}{z}$

$Ca^{+2} < K^+ < Cl^- < S^{-2}$: size

Z : 20 19 17 18

43. Cobalt chloride when dissolved in water forms pink colored complex $\underline{X}$ which has octahedral geometry. This solution on treating with conc HCl forms deep blue complex, $\underline{Y}$ which has a $\underline{Z}$ geometry. X, Y and Z, respectively, are

(1) $X = [Co(H_2O)_6]^{2+}, Y = [CoCl_4]^{2-}, Z =$ Tetrahedral

(2) $X = [Co(H_2O)_6]^{2+}, Y = [CoCl_6]^{3-}, Z =$ Octahedral

(3) $X = [Co(H_2O)_4Cl_2]^+, Y = [CoCl_4]^{2-}, Z =$ Tetrahedral

(4) $X = [Co(H_2O)_6]^{3+}, Y = [CoCl_6]^{3-}, Z =$ Octahedral

Sol. **1**

$$CoCl_2 + H_2O \longrightarrow \underset{\text{Pink}}{[Co(H_2O)_6]^{2+}} \xrightarrow{\text{conc. HCl}} \underset{\text{Blue Tetrahderal}}{[CoCl_4]^{2-}}$$

44. H_2O_2 acts as a reducing agent in

(1) $2NaOCl + H_2O_2 \rightarrow 2NaCl + H_2O + O_2$

(2) $Na_2S + 4H_2O_2 \rightarrow Na_2SO_4 + 4H_2O$

(3) $2Fe^{2+} + 2H^+ + H_2O_2 \rightarrow 2Fe^{3+} + 2H_2O$

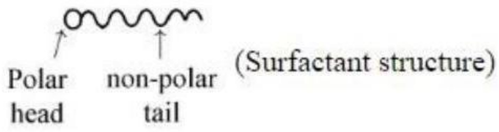
(4) $Mn^{2+} + 2H_2O_2 \rightarrow MnO_2 + 2H_2O$

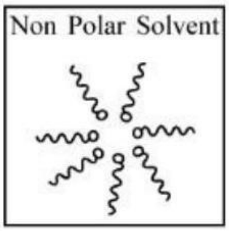
Sol. **1**

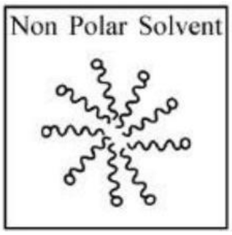
$$2NaO\overset{+1}{Cl} + H_2O_2 \longrightarrow 2Na\overset{-1}{Cl} + H_2O + O_2$$

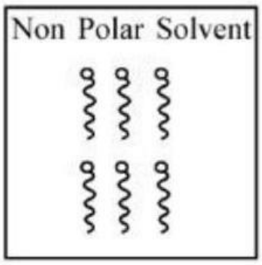
H_2O_2 acts as reducing agent.

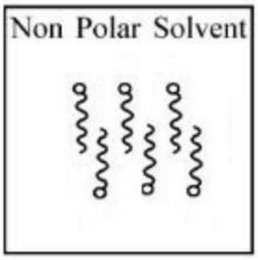
45. Adding surfactants in non polar solvent, the micelles structure will look like



(a) 

(b) 

(c) 

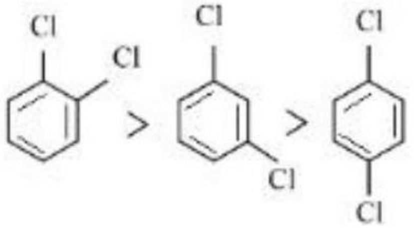
(d) 

(1) a (2) d (3) b (4) c

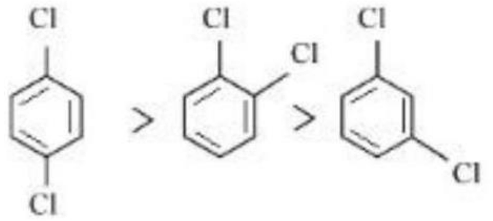
Sol. **1**

Non polar end will be towards non polar solvent

46. The correct order of melting points of dichlorobenzenes is



(1)



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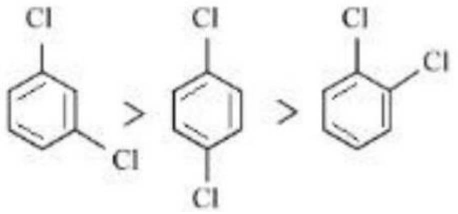

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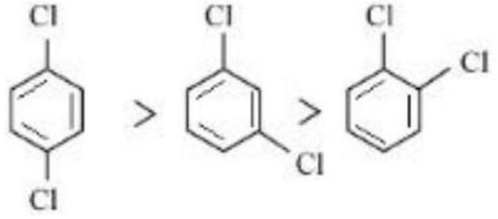
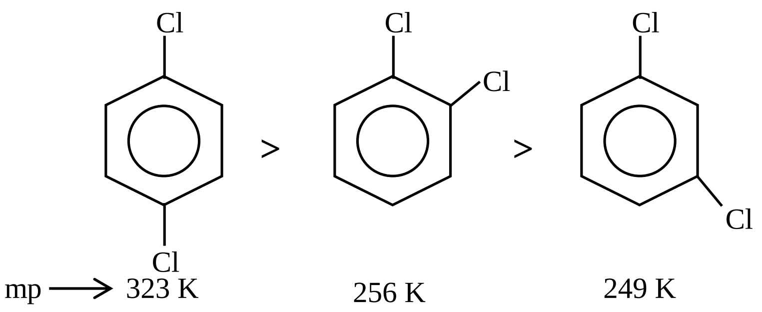


(4)

Sol. **2**



47. The correct order of basicity of oxides of vanadium is

(1) $V_2O_5 > V_2O_4 > V_2O_3$ (2) $V_2O_4 > V_2O_3 > V_2O_5$

(3) $V_2O_3 > V_2O_5 > V_2O_4$ (4) $V_2O_3 > V_2O_4 > V_2O_5$

Sol. **4**

Leaser is charge on canter atom more will be the basicity.

48. Which of the following artificial sweeteners has the highest sweetness value in comparison to cane sugar ?

(1) Sucralose (2) Aspartame (3) Alitame (4) Saccharin

Sol. **3**

Alitame has 2000 has times more sweetner as compare to cane sugar.

49. Which one of the following statements is correct for electrolysis of brine solution?

(1) Cl_2 is formed at cathode (2) O_2 is formed at cathode

(3) H_2 is formed at anode (4) $OH^-$is formed at cathode

Sol. **4**

Brine sol^n gives H_2/OH^- at cathode & Cl_2 at anode.

50. Which transition in the hydrogen spectrum would have the same wavelength as the Balmer type transition from $n = 4$ to $n = 2$ of He^+spectrum

(1) $n = 2$ to $n = 1$ (2) $n = 1$ to $n = 2$ (3) $n = 3$ to $n = 4$ (4) $n = 1$ to $n = 3$

Sol. **1**

$$\lambda_H = \lambda_{He^+}$$

$$R_H \times (1)^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = R_H \times (2)^2\left(\frac{1}{(2)^2} - \frac{1}{(4)^2}\right)$$

$$\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = \left(\frac{4}{4}\right) - \left(\frac{4}{16}\right)$$

$$\frac{1}{n_1^2} - \frac{1}{n_2^2} = \frac{1}{1} - \frac{1}{4}$$

$n_1 = 1 : n_2 = 2$ for H-atom

SECTION B

51. The oxidation state of phosphorus in hypophosphoric acid is +

Sol. Hypophosphoric acid is $H_4P_2O_6$ oxidation state of P is +4.

52. The enthalpy change for the conversion of $\frac{1}{2}Cl_2(g)$ to Cl^-(aq) is (-) $kJmol^{-1}$ (Nearest integer)

Given : $\Delta_{dis} H^{\ominus}_{Cl_{2(g)}} = 240 \text{ kJ mol}^{-1}, \Delta_{eg} H^{\ominus}_{Cl_{(g)}} = -350 \text{ kJ mol}^{-1}$, $\Delta_{hyd} H^{\ominus}_{Cl^-_{(g)}} = -380 \text{ kJ mol}^{-1}$

Sol. **610**

$$\begin{tikzcd}
\frac{1}{2}\,Cl_2 \arrow[r] \arrow[d, "\frac{1}{2}\ BE"'] & Cl^{-}\ aq. \\
Cl_{(g)} \arrow[r, "\Delta H_{eg}.\ Of\ Cl_{(g)}"'] & Cl_{(g)} \arrow[u, "\Delta H_{Hyd.}\ Of\ Cl_{(g)}"']
\end{tikzcd}$$

$$\Delta H_{\gamma}^{\circ} = \frac{1}{2} \times BE + \Delta H_{eg} + \Delta H_{Hyd}$$

$$= \frac{1}{2} \times 240 + (-350) + (-380)$$

$\Rightarrow$ 120 – 350 – 380

$\Rightarrow$ –610

53. The logarithm of equilibrium constant for the reaction $Pd^{2+} + 4Cl^{-} \rightleftharpoons PdCl_4^{2-}$ is (Nearest integer)

Given : $\frac{2.303RT}{F} = 0.06\ V$

$Pd^{2+}_{(aq)} + 2e^{-} \rightleftharpoons Pd(s)\ E^{\ominus} = 0.83\ V$

$PdCl_4^{2-}(aq) + 2e^{-} \rightleftharpoons Pd(s) + 4Cl^{-}(aq)\ E^{\theta} = 0.65\ V$

Sol. **6**

$\Delta G° = -RT\ell nK$

$-nFE°_{cell} = -RT \times 2.303\ (\log_{10}K)$ ....(1)

Net reaction → Pd^{2+} (aq.) + $4Cl^{-}$ (aq.) $\rightleftharpoons$ $PdCl_4^{2-}$ (aq.)

$E°_{cell} = E°_{cathod} - E°_{anode}$

$E°_{cell} = 0.83 - 0.65$

From equation (1)

Also n = 2

logK = 6

54. On complete combustion, 0.492 g of an organic compound gave 0.792 g of CO_2.
The % of carbon in the organic compound is (Nearest integer)

Sol. **44**

44 gm of CO_2 contains 12 g carbon.

0.792 gm of CO_2 contains $\frac{0.792 \times 12}{44}$ g of carbon

% of carbon = $\frac{0.216}{0.492} \times 100$

= 43.9% = 44%

55. Zinc reacts with hydrochloric acid to give hydrogen and zinc chloride. The volume of hydrogen gas produced at STP from the reaction of 11.5 g of zinc with excess HCl is L (Nearest integer)
(Given : Molar mass of Zn is $65.4\ g\ mol^{-1}$ and Molar volume of H_2 at $STP = 22.7\ L$)

Sol. **4**

$$Zn + 2HCl \longrightarrow ZnCl_2 + H_{2.}$$

No. of moles of Zn = $\frac{11.5}{65.3}$ = No. of moles of H_2

No. of H_2 liberated = 0.176 × 22.7 Lt.

= 3.99 L = 4 Lt.

56. $A \rightarrow B$

The rate constants of the above reaction at $200 \mathrm{~K}$ and $300 \mathrm{~K}$ are $0.03 \mathrm{~min}^{-1}$ and $0.05 \mathrm{~min}^{-1}$ respectively. The activation energy for the reaction is J (Nearest integer) (Given : $\ln 10=2.3$

$\mathrm{R}=8.3 \mathrm{~J} \mathrm{~K}^{-1} \mathrm{~mol}^{-1}$

$\log 5=0.70$

$\log 3=0.48$

$\log 2=0.30)$

Sol. 2520

$$\text{In}\left(\frac{K_2}{K_1}\right)=\frac{Ea}{R}\left[\frac{1}{T_1}-\frac{1}{T_2}\right]$$

$$\text{Log}\left(\frac{0.05}{0.03}\right)=\frac{Ea}{2.3 \times 8.3}\left[\frac{1}{200}-\frac{1}{300}\right]$$

$$[0.70-0.48]=\frac{Ea}{2.3 \times 8.3}\left[\frac{300-200}{300 \times 200}\right]$$

$$0.22=\frac{Ea}{2.3 \times 8.3}\left[\frac{1}{600}\right]$$

Ea = 0.22 × 2.3 × 8.3 × 600

= 2519.88 J

≈ 2520

57. For reaction: $\mathrm{SO}_2(\mathrm{~g})+\frac{1}{2} \mathrm{O}_2(\mathrm{~g}) \rightleftharpoons \mathrm{SO}_3(\mathrm{~g})$

$\mathrm{K}_{\mathrm{p}}=2 \times 10^{12}$ at $27^{\circ} \mathrm{C}$ and 1 atm pressure. The K_{c} for the same reaction is $\times 10^{13}$. (Nearest integer)

(Given $\mathrm{R}=0.082 \mathrm{~L}$ atm $\mathrm{K}^{-1} \mathrm{~mol}^{-1}$)

Sol. **1**

$K_C = 1 \times 10^{13}$

$\mathrm{SO}_2(\mathrm{~g})+\frac{1}{2} \mathrm{O}_2 \rightleftharpoons \mathrm{SO}_3(\mathrm{g})$

$\Delta n=\frac{-1}{2}$

$K_P = K_C (RT)^{\Delta ng}$

$2 \times 10^{12} = K_C (0.082 \times 300)^{-1/2}$

$K_C = 1 \times 10^{13}$

$K_P = 2 \times 10^{12}$

P = 1 atm

T = 27°C

58. The total pressure of a mixture of non-reacting gases $X(0.6 \mathrm{~g})$ and $Y(0.45 \mathrm{~g})$ in a vessel is $740 \mathrm{~mm}$ of Hg. The partial pressure of the gas X is mm of Hg.

(Nearest Integer)

(Given : molar mass $\mathrm{X}=20$ and $\mathrm{Y}=45 \mathrm{~g} \mathrm{~mol}^{-1}$)

Sol. **555**

Number of moles of gas $X=\frac{0.6}{20}=0.03$

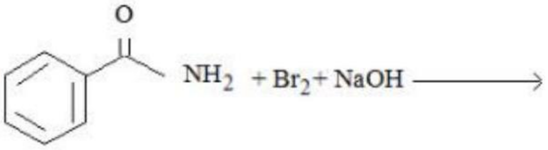
Number of moles of gas $Y=\frac{0.45}{45}=0.01$

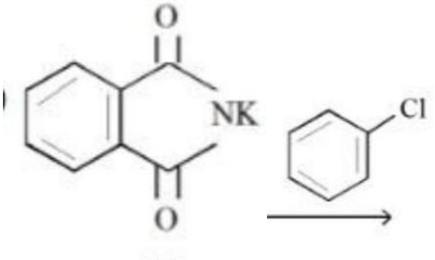
Total number of moles = 0.03 + 0.01 = 0.04 mole

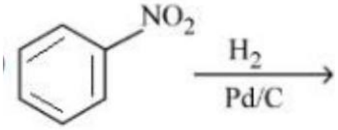
Partial pressure of gas X = Mole fraction × Total pressure

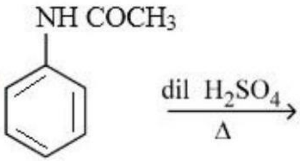
$$=\frac{0.03}{0.04} \times 740=555$$

59. How many of the transformations given below would result in aromatic amines ?

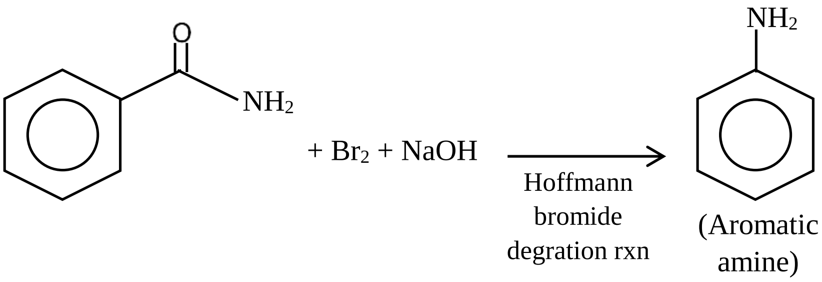
(1) 

(2) 

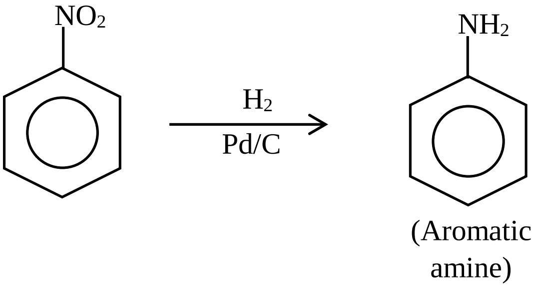
(3) 

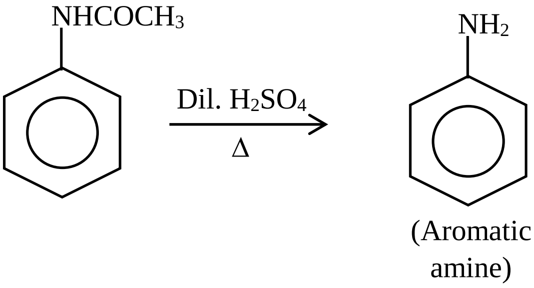
(4) 

Sol. **3**

(1) 

(2) In Gabriel phthalimide synthesis chloro-benzene is poor substrok for S_{N_2} , Hence reaction will not observed.

(3) 

(4) 

60. At $27^\circ\mathrm{C}$, a solution containing $2.5\ \mathrm{g}$ of solute in $250.0\ \mathrm{mL}$ of solution exerts an osmotic pressure of $400\ \mathrm{Pa}$. The molar mass of the solute is gmol^{-1} (Nearest integer)

(Given : $\mathrm{R} = 0.083\ \mathrm{L}_{\mathrm{bar}}\ \mathrm{K}^{-1}\ \mathrm{mol}^{-1}$)

Sol. **62250**

π = CRT

$$\frac{400\mathrm{Pa}}{10^5} = \frac{\frac{2.5\mathrm{g}}{\mathrm{M}_\circ}}{250/1000} \times 0.083\frac{\mathrm{L-bar}}{\mathrm{Kmol}} \times 300\,\mathrm{K}$$

$\mathrm{M}_\circ$ = 62250

Mathematics

SECTION - A

61. If the maximum distance of normal to the ellipse $\frac{x^2}{4}+\frac{y^2}{b^2}=1, b<2$, from the origin is 1 , then the eccentricity of the ellipse is :

(1) $\frac{1}{2}$ (2) $\frac{\sqrt{3}}{4}$ (3) $\frac{\sqrt{3}}{2}$ (4) $\frac{1}{\sqrt{2}}$

Sol.

Normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at point (a cosθ, b sinθ) is $\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2$

Its distance from origin is

$$d=\frac{|a^2-b^2|}{\sqrt{a^2\sec^2\theta+b^2\operatorname{cosec}^2\theta}}$$

$$d=\frac{|a^2-b^2|}{\sqrt{a^2+b^2+2ab+(a\ \tan\theta-b\ \cot\theta)^2}}$$

$$d\frac{|(a-b)(a+b)|}{\sqrt{a^2+b^2+2ab+(a\ \tan\theta-b\ \tan\theta)^2}}$$

$$d_{max}=\frac{|(a-b)(a+b)|}{a+b}=|a-b|$$

$\because$ $d_{max}=1$

$|2-b|=1$

$2-b=1$ $[\because b<2]$

$$\boxed{b=1}$$

$$\text{Eccentricity}=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{1}{4}}=\frac{\sqrt{3}}{2}$$

$$\Rightarrow \boxed{e=\frac{\sqrt{3}}{2}}$$

62. Let a differentiable function f satisfy $f(x)+\int_3^x\frac{f(t)}{t}dt=\sqrt{x+1}, x\ge 3$. Then $12f(8)$ is equal to :

(1) 34 (2) 1 (3) 17 (4) 19

Sol.

$$f(x)+\int_3^x\frac{f(t)}{t}dt=\sqrt{x+1},\ x\ge 3$$

Differentiate both side w.r.t. x

$$f^1(x)+\frac{f(x)}{x}=\frac{1}{2\sqrt{x+1}}$$

Above eqn. is linear differential equation

I.f. = $e^{\int \frac{1}{x}dx} = e^{\ln x} = x$

Solution is

$$f(x)\cdot x = \int \frac{x}{2\sqrt{x+1}}dx + C$$

$$f(x)\cdot x = \frac{1}{2}\int\left(\frac{x+1}{\sqrt{x+1}} - \frac{1}{\sqrt{x+1}}\right)dx + C$$

$$f(x)\cdot x = \frac{1}{2}\int\left(\sqrt{x+1} - \frac{1}{\sqrt{x+1}}\right)dx + C$$

$$f(x)\cdot x = \frac{1}{2}\left[\frac{2}{3}(x+1)^{3/2} - 2\sqrt{x+1}\right] + C$$

$\because$ f(3) = 2

than

$$2.3 = \frac{1}{2}\left[\frac{2}{3}\times 8 - 2\times 2\right] + C$$

$$6 = \frac{1}{2}\left[\frac{16}{3} - 4\right] + C$$

$$6 = \frac{2}{3} + C$$

$$\boxed{C = \frac{16}{3}}$$

$$f(x).\ x = \frac{1}{2}\left[\frac{2}{3}(x+1)^{3/2} - 2\sqrt{x+1}\right] + \frac{16}{3}$$

Put x = 8

$$f(8)\cdot 8 = \frac{1}{2}\left[\frac{2}{3}\times 27 - 2\times 3\right] + \frac{16}{3}$$

$$f(8)\cdot 8 = \frac{1}{2}[12] + \frac{16}{3}$$

$$f(8)\cdot 8 = 6 + \frac{16}{3} = \frac{34}{3}$$

$$\boxed{12f(8) = 17}$$

63. For all $z \in C$ on the curve $C_1: |z| = 4$, let the locus of the point $z + \frac{1}{z}$ be the curve C_2. Then :

(1) the curve C_1 lies inside C_2

(2) the curve C_2 lies inside C_1

(3) the curves C_1 and C_2 intersect at 4 points

(4) the curves C_1 and C_2 intersect at 2 points

Sol.

$C_1 : |z| = 4$ then $z\overline{z} = 16$

$$z + \frac{1}{z} = z + \frac{\overline{z}}{16}$$

$$= x + iy + \frac{x - iy}{16}$$

$$z + \frac{1}{z} = \frac{17x}{16} + i\frac{15y}{16}$$

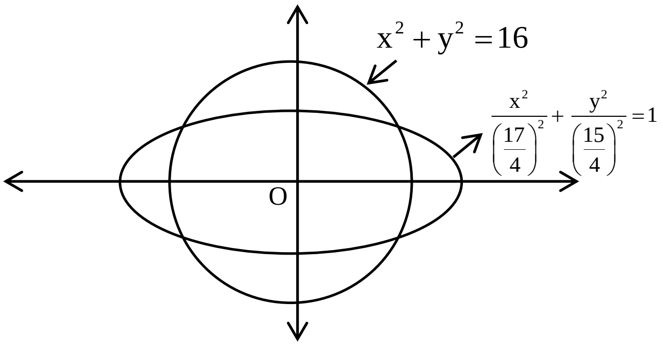
Let $X = \frac{17x}{16}, \quad Y = \frac{15}{16}y$

$$\frac{X}{\left(\frac{17}{16}\right)} = x, \ \frac{Y}{\left(\frac{15}{16}\right)} = y$$

$\because x^2 + y^2 = 16$

$$\frac{X^2}{\left(\frac{17}{16}\right)^2} + \frac{Y^2}{\left(\frac{15}{16}\right)^2} = 16$$

$$\Rightarrow C_2 : \frac{x^2}{\left(\frac{17}{4}\right)^2} + \frac{y^2}{\left(\frac{15}{4}\right)^2} = 1 \quad \text{(Ellipse)}$$



Curve C_1 and C_2 intersect at 4 point.

64. $y = f(x) = \sin^3\left(\frac{\pi}{3}\left(\cos\left(\frac{\pi}{3\sqrt{2}}(-4x^3 + 5x^2 + 1)^{\frac{3}{2}}\right)\right)\right)$. Then, at $x = 1$,

(1) $\sqrt{2}y' - 3\pi^2 y = 0$ (2) $y' + 3\pi^2 y = 0$ (3) $2y' + 3\pi^2 y = 0$ (4) $2y' + \sqrt{3}\pi^2 y = 0$

Sol.

$$y = f(x) = \sin^3\left(\frac{\pi}{3}\left(\cos\left(\frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{3/2}\right)\right)\right)$$

Let $g(x) = \frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{3/2}$

$g(1) = \frac{2\pi}{3}$

$$y = \sin^3\left(\frac{\pi}{3}\cos(g(x))\right)$$

Differentiate w.r.t. x

$$y' = 3\sin^2\left(\frac{\pi}{3}\cos(g(x))\right)\times\cos\left(\frac{\pi}{3}\cos(g(x))\right)\times\frac{\pi}{3}\left(-\sin g(x)\right)g'(x)$$

$$\because g^1(x) = \frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{1/2}\left(-12x^2+10x\right)$$

$$g^1(1) = \frac{\pi}{2\sqrt{2}}\left(\sqrt{2}\right)\left(-2\right) = -\pi$$

$$y^1(1) = \frac{3}{4}\times\frac{\sqrt{3}}{2}\times\frac{\pi}{3}\left(\frac{-\sqrt{3}}{2}\right)\left(-\pi\right) = \frac{3\pi^2}{16}$$

$$\boxed{y^1(1) = \frac{3\pi^2}{16}}$$

$$y(1) = \sin^{3.}\left(\frac{\pi}{3}\cos\frac{2\pi}{3}\right) = \frac{-1}{8}$$

$$\boxed{2y^1(1)+3\pi^2 y(1) = 0}$$

65. A wire of length 20 m is to be cut into two pieces. A piece of length l_1 is bent to make a square of area A_1 and the other piece of length l_2 is made into a circle of area A_2. If $2A_1 + 3A_2$ is minimum then $(\pi l_1) : l_2$ is equal to :

(1) 1:6 (2) 6: 1 (3) 3: 1 (4) 4: 1

Sol.

Total length of wire = 20 m

area of square (A_1) = $\left(\frac{\ell_1}{4}\right)^2$

area of circle (A_2) = $\pi\left(\frac{\ell_2}{2\pi}\right)^2$

Let S = $2A_1 + 3A_2$

$$S = \frac{\ell_1^2}{8} + \frac{3\ell_2^2}{4\pi}$$

$\because \ \ell_1 + \ell_2 = 20$ then

$$1 + \frac{d\ell_2}{d\ell_1} = 0$$

$$\frac{d\ell_2}{d\ell_1} = -1$$

$$\frac{ds}{d\ell_1} = \frac{\ell_1}{4} + \frac{6\ell_2}{4\pi} \cdot \frac{d\ell_2}{d\ell_1} = 0$$

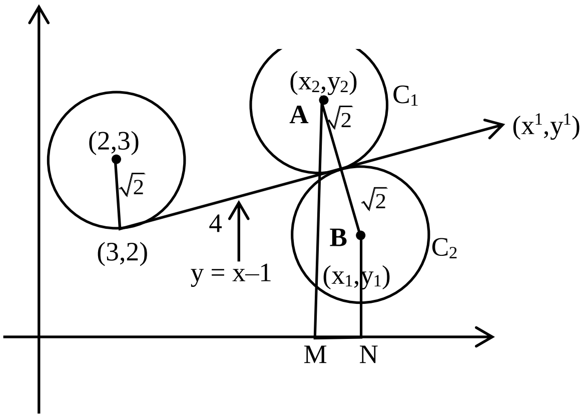
$$= \frac{\ell_1}{4} = \frac{6\ell_2}{4\pi}$$

$$= \frac{\pi\ell_1}{\ell_2} = \frac{6}{1}$$

66. Let a circle C_1 be obtained on rolling the circle $x^2 + y^2 - 4x - 6y + 11 = 0$ upwards 4 units on the tangent T to it at the point $(3,2)$. Let C_2 be the image of C_1 in T. Let A and B be the centers of circles C_1 and C_2 respectively, and M and N be respectively the feet of perpendiculars drawn from A and B on the x-axis. Then the area of the trapezium AMNB is :

(1) $4(1 + \sqrt{2})$ (2) $3 + 2\sqrt{2}$ (3) $2(1 + \sqrt{2})$ (4) $2(2 + \sqrt{2})$

Sol.



(x', y') point lies on line y = x – 1 have distance 4 unit from (3, 2).

$$x' = \frac{4}{\sqrt{2}} + 3 \quad = 2\sqrt{2} + 3$$

$$y' = \frac{4}{\sqrt{2}} + 2 = 2\sqrt{2} + 2$$

Slope of line AB is –1. i.e. = $\tan\theta = -1$ then $\sin\theta = \frac{1}{\sqrt{2}}$, $\cos\theta = -\frac{1}{\sqrt{2}}$

for point A and B

$$x = \pm\sqrt{2}\left(\frac{-1}{\sqrt{2}}\right)+\left(2\sqrt{2}+3\right)$$

$$y = \pm\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)+\left(2\sqrt{2}+2\right)$$

for point A we take +ve sign

$$(x_2, y_2) = \left(2\sqrt{2}+2,\ 2\sqrt{2}+3\right)$$

for point B we take –ve sign

$$(x_1, y_1) = \left(2\sqrt{2}+4,\ 2\sqrt{2}+1\right)$$

MN = $|x_2 - x_1| = 2$

AM + BN = $2\sqrt{2}+3+2\sqrt{2}+1=4+4\sqrt{2}$

area of trapezium = $\frac{1}{2}\times 2\times\left(4+4\sqrt{2}\right)$

$$= 4\left(1+\sqrt{2}\right)$$

67. A bag contains 6 balls. Two balls are drawn from it at random and both are found to be black. The probability that the bag contains at least 5 black balls is

(1) $\frac{3}{7}$ (2) $\frac{5}{7}$ (3) $\frac{5}{6}$ (4) $\frac{2}{7}$

Sol. $\text{Probability} = \dfrac{{}^3C_2 + {}^6C_2}{{}^2C_2 + {}^3C_2 + {}^4C_2 + {}^5C_2 + {}^6C_2}$

$$= \frac{10+15}{1+3+6+10+15}$$

$$= \frac{5}{7}$$

68. Let $y = f(x)$ represent a parabola with focus $\left(-\frac{1}{2}, 0\right)$ and directrix $y = -\frac{1}{2}$.

Then $S = \left\{x \in \mathbb{R}: \tan^{-1}(\sqrt{f(x)}) + \sin^{-1}(\sqrt{f(x)+1}) = \frac{\pi}{2}\right\}$:

(1) contains exactly two elements (2) contains exactly one element

(3) is an empty set (4) is an infinite set

Sol. equation of parabola which have focus $\left(-\frac{1}{2},\ 0\right)$ and directrix y = $-\frac{1}{2}$ is

$$\left(x+\frac{1}{2}\right)^2 = \left(y+\frac{1}{4}\right)$$

y = f(x) = $(x^2 + x)$

$\because\ S = \left\{x \in R : \tan^{-1}\left(\left(\sqrt{f(x)}\right) + \sin^{-1}\left(\sqrt{f(x)+1}\right) = \frac{\pi}{2}\right)\right\}$

$\tan^{-1}\left(\sqrt{f(x)}\right) + \sin^{-1}\left(\sqrt{f(x)+1}\right) = \frac{\pi}{2}$

$f(x) \geq 0$ & $\sqrt{f(x)+1}$ can not greater then 1, so f(x) must be 0

i.e. f(x) = 0

$\Rightarrow\ x^2 + x = 0$

$x(x+1) = 0$

$x = 0,\ x = -1$

S contain 2 element.

69. Let $\vec{a} = 2\hat{\imath} + \hat{\jmath} + \hat{k}$, and $\vec{b}$ and $\vec{c}$ be two nonzero vectors such that $|\vec{a} + \vec{b} + \vec{c}| = |\vec{a} + \vec{b} - \vec{c}|$ and $\vec{b} \cdot \vec{c} = 0$. Consider the following two statements:

(A) $|\vec{a} + \lambda\vec{c}| \geq |\vec{a}|$ for all $\lambda \in \mathbb{R}$.

(B) $\vec{a}$ and $\vec{c}$ are always parallel.

Then.

(1) both (A) and (B) are correct
(2) only (A) is correct
(3) neither (A) nor (B) is correct
(4) only (B) is correct

Sol.

$\left|\vec{a} + \vec{b} + \vec{c}\right| = \left|\vec{a} + \vec{b} - \vec{c}\right|,\ \vec{b}.\vec{c} = 0$

$\left|\vec{a} + \vec{b} + \vec{c}\right|^2 = \left|\vec{a} + \vec{b} - \vec{c}\right|^2$

$\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + \left|\vec{c}\right|^2 + 2\vec{a}.\vec{b} + 2\vec{b}.\vec{c} + 2\vec{a}.\vec{c}$

$= \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + \left|\vec{c}\right|^2 + 2\vec{a}.\vec{b} - 2\vec{b}.\vec{c} - 2\vec{a}.\vec{c}$

$2\vec{a}.\vec{b} + 2\vec{b}.\vec{c} + 2\vec{a}.\vec{c} = 2\vec{a}.\vec{b} - 2\vec{b}.\vec{c} - 2\vec{a}.\vec{c}$

$\vec{a}.\vec{b} + \vec{a}.\vec{c} = \vec{a}.\vec{b} - \vec{a}.\vec{c}$

$\vec{a}.\vec{c} = 0$ (B is incorrect)

$\left|\vec{a} + \lambda\vec{c}\right|^2 \geq \left|\vec{a}\right|^2$

$\left|\vec{a}\right|^2 + \lambda^2\left|\vec{c}\right|^2 + 2\lambda\vec{a}\cdot\vec{c} \geq \left|\vec{a}\right|^2$

$= \lambda^2 c^2 \geq 0$

True $\forall \lambda \in R$ (A is correct)

70. The value of $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{(2+3\sin x)}{\sin x(1+\cos x)} dx$ is equal to

(1) $\frac{10}{3} - \sqrt{3} - \log_e \sqrt{3}$

(2) $\frac{7}{2} - \sqrt{3} - \log_e \sqrt{3}$

(3) $-2 + 3\sqrt{3} + \log_e \sqrt{3}$

(4) $\frac{10}{3} - \sqrt{3} + \log_e \sqrt{3}$

Sol. **(4)**

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{(2+3\sin x)}{\sin x(1+\cos x)} dx$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{2}{\sin x + \sin x \cos x} dx + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{3}{1+\cos x} dx$$

$= I_1 + I_2$

$$I_1 = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{2dx}{\sin x(1+\cos x)} \quad = 2\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\left(1+\tan^2 \frac{x}{2}\right)dx}{2\tan\frac{x}{2} \times \left(1 + \frac{1-\tan^2 \frac{x}{2}}{1+\tan^2 \frac{x}{2}}\right)}$$

$$= 2\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\left(1+\tan^2 \frac{x}{2}\right)\left(1+\tan^2 \frac{x}{2}\right)dx}{2\tan\frac{x}{2} \times 2}$$

$$= 2\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sec^2 \frac{x}{2}\left(1+\tan^2 \frac{x}{2}\right)}{4\tan\frac{x}{2}} dx$$

Let, $\tan\frac{x}{2} = t$ then $\sec^2\frac{x}{2} \times \frac{1}{2}dx = dt$

$$= 2\int_{\frac{1}{\sqrt{3}}}^{1} \frac{1+t^2}{2t} dt$$

$$= \left[\ell nt + \frac{t^2}{2}\right]_{\frac{1}{\sqrt{3}}}^{1}$$

$$= \left[\frac{1}{2} - \ell n\frac{1}{\sqrt{3}} - \frac{1}{6}\right]$$

$$I_1 = \left[\ell n\sqrt{3} + \frac{1}{3}\right]$$

$$I_2 = 3\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{dx}{1+\cos x} = 3\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1-\cos x}{\sin^2 x} dx$$

$$I_2 = 3\left[\cos ecx - \cot x\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} = 3-\sqrt{3}$$

$$I_1 + I_2 = \ln\sqrt{3} + \frac{1}{3} + 3 - \sqrt{3}$$

$$= \frac{10}{3} + \ln\sqrt{3} - \sqrt{3}$$

71. Let the shortest distance between the lines $L: \frac{x-5}{-2} = \frac{y-\lambda}{0} = \frac{z+\lambda}{1}, \lambda \geq 0$ and $L_1: x+1 = y-1 = 4-z$ be $2\sqrt{6}$. If (α, β, γ) lies on L, then which of the following is NOT possible ?

(1) $\alpha - 2\gamma = 19$ (2) $2\alpha + \gamma = 7$ (3) $2\alpha - \gamma = 9$ (4) $\alpha + 2\gamma = 24$

Sol. **(4)**

Let $\vec{b}_1 =< -2,0,1 > \quad \vec{a}_1 = (5,\lambda,-\lambda)$

$\vec{b}_2 =< 1,1,-1 \ > \vec{a}_2 = (-1,1,4)$

Normal vector of both line is $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix}$

$\hat{i}(-1) - \hat{j}(1) + \hat{k}(-2)$

$\vec{b}_1 \times \vec{b}_2 =< -1,-1,-2 >$

$\vec{a}_1 - \vec{a}_2 =< 6, \lambda-1, -\lambda-4 >$

Shortest distance d = $\left| \frac{(\vec{a}_2 - \vec{a}_1) \times (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$

$$2\sqrt{6} = \left| \frac{< 6, \lambda-1, -\lambda-4 > \times < -1,-1,-2 >}{\sqrt{(1)^2 + (1)^2 + (2)^2}} \right|$$

$$12 = |-6 - \lambda + 1 + 2\lambda + 8|$$

$$|\lambda + 3| = 12$$

$$\lambda = 9, \text{-}15$$

$$\lambda = 9 (\because \lambda \geq 0)$$

$\because$ (α, β, γ) lies on line L then

$$\frac{\alpha-5}{-2} = \frac{\beta-9}{0} = \frac{\gamma+9}{1} = K$$

$\alpha = 5 - 2K$, $\beta = 9K$, $\gamma = -9 + K$

$\alpha + 2\gamma$, $= 5 - 2K - 18 + 2K = -13 \neq 24$

Therefore $\alpha + 2\gamma = 24$ is not possible.

72. For the system of linear equations

$x + y + z = 6$

$\alpha x + \beta y + 7z = 3$

$x + 2y + 3z = 14$

which of the following is NOT true ?

(1) If $\alpha = \beta$ and $\alpha \neq 7$, then the system has a unique solution

(2) If $\alpha = \beta = 7$, then the system has no solution

(3) For every point $(\alpha, \beta) \neq (7,7)$ on the line $x - 2y + 7 = 0$, the system has infinitely many solutions

(4) There is a unique point (α, β) on the line $x + 2y + 18 = 0$ for which the system has infinitely many solutions

Sol. **(3)**

$x + y + z = 6$ … (1)

$\alpha x + \beta y + 7z = 3$ … (2)

$x + 2y + 3z = 14$ … (3)

equation (3) – equation (1)

$y + 2z = 8$

$y = 8 - 2z$

From (1) $x = -2 + z$

Value of x and y put in equation (2)

$\alpha(-2 + z) + \beta(8 - 2z) + 7z = 3$

$-2\alpha + \alpha z + 8\beta - 2\beta z + 7z = 3$

$(\alpha - 2\beta + 7) z = 2\alpha - 8\beta + 3$

if $\alpha - 2\beta + 7 \neq 0$ then system has unique solution

if $(\alpha - 2\beta + 7 = 0)$ and $2\alpha - 8\beta + 3 \neq 0$ then system has no solution

if $(\alpha - 2\beta + 7 = 0)$ and $2\alpha - 8\beta + 3 = 0$ then system has infinite solution

73. If the domain of the function $f(x) = \frac{[x]}{1+x^2}$, where $[x]$ is greatest integer $\leq x$, is $[2,6)$, then its range is

(1) $\left(\frac{5}{26}, \frac{2}{5}\right]$

(2) $\left(\frac{5}{37}, \frac{2}{5}\right] - \left\{\frac{9}{29}, \frac{27}{109}, \frac{18}{89}, \frac{9}{53}\right\}$

(3) $\left(\frac{5}{37}, \frac{2}{5}\right]$

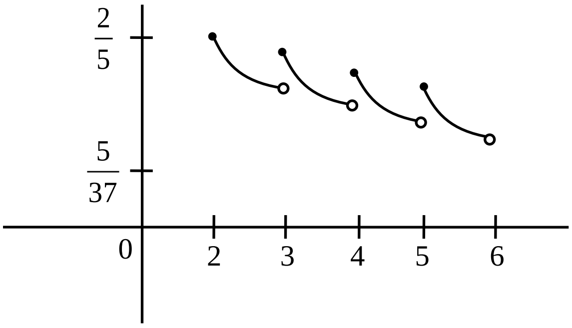
(4) $\left(\frac{5}{26}, \frac{2}{5}\right] - \left\{\frac{9}{29}, \frac{27}{109}, \frac{18}{89}, \frac{9}{53}\right\}$

Sol. **(3)**

$f(x) = \frac{[x]}{1+x^2}$, $x = \in [2, 6]$

$$f(x)=\begin{cases}\dfrac{2}{1+x^2} & x\in[2,3)\\ \dfrac{3}{1+x^2} & x\in[3,4)\\ \dfrac{4}{1+x^2} & x\in[4,5)\\ \dfrac{5}{1+x^2} & x\in[5,6)\end{cases}$$

$\because f(x)$ is $\downarrow$ in $x\in[2, 6)$



range is $\left(\dfrac{5}{37},\dfrac{2}{5}\right]$

74. Let R be a relation on N×N defined by $(a, b)R(c, d)$ if and only if $ad(b-c)=bc(a-d)$. Then R is

(1) transitive but neither reflexive nor symmetric

(2) symmetric but neither reflexive nor transitive

(3) symmetric and transitive but not reflexive

(4) reflexive and symmetric but not transitive

Sol. **(2)**

(a, b) R (c, d) $\Leftrightarrow$ ad(b – c) = bc(a – d)

For reflexive

(a, b) R (a, b)

$\Rightarrow$ ab (b – a) $\neq$ ba(a – b)

R is not reflexive

For symmetric:

(a,b) R(c,d) $\Rightarrow$ ad(b –c) = bc (a –d)

then we check

(c, d) R (a, b) $\Rightarrow$ cb(d – a) = ad(c – b)

$\Rightarrow$ cb(a – d) = ad(b – c)

R is symmetric :

For transitive:

$\because$ (2,3) R (3,2) and (3,2) R (5,30)

But (2,3) is not related to (5,30)

R is not transitive.

75. $(S1)(p \Rightarrow q) \vee (p \wedge (\sim q))$ is a tautology

$(S2)((\sim p) \Rightarrow (\sim q)) \wedge ((\sim p) \vee q)$ is a contradiction.

Then

(1) both (S1) and $(S2)$ are correct (2) only ($S1$) is correct

(3) only (S2) is correct (4) both (S1) and (S2) are wrong

Sol. **(2)**

S_1 : $(P \Rightarrow q)$ V $(P \wedge (\sim q))$

P	q	P⇒q	~q	PΛ~q	(P ⇒ q) V (P ∧ ~q)
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	T	F	T

S_1 is a tautology

S_2 : $((\sim P) \Rightarrow (\sim q))$ Λ $((\sim P)$ V$q)$

~P	~q	~P⇒~q	~P v q	((~P) ⇒ (~q)) Λ (~P) vq)
F	F	T	T	T
F	T	T	F	F
T	F	F	T	F
T	T	T	T	T

S_2 is not a contradiction

76. If the sum and product of four positive consecutive terms of a G.P., are 126 and 1296 , respectively, then the sum of common ratios of all such GPs is

(1) 7 (2) 3 (3) $\frac{9}{2}$ (4) 14

Sol. **(1)**

Four term of G.P. $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$

$$\frac{a}{r^3} + \frac{a}{r} + ar + ar^3 = 126$$

$$\frac{a}{r^3} \cdot \frac{a}{r} \cdot ar \cdot ar^3 = 1296$$

$a^4 = 1296$

$a = 6$

$$\frac{6}{r^3} + \frac{6}{r} + 6r + 6r^3 = 126$$

$$\left(r+\frac{1}{r}\right)+r^3+\frac{1}{r^3}=21$$

$$\left(r+\frac{1}{r}\right)+\left(r+\frac{1}{r}\right)^3-3\left(r+\frac{1}{r}\right)=21$$

Let $r + \frac{1}{r} = t$

$t^3 - 2t = 21$

$\Rightarrow t = 3$

$r + \frac{1}{r} = 3$

$r^2 - 3r + 1= 0$

$r = \frac{3 \pm \sqrt{9-4}}{2}$

$r = \frac{3 \pm \sqrt{5}}{2}$

Sum of common ratio = $\frac{9}{4}+\frac{5}{4}+\frac{3\sqrt{5}}{2}+\frac{9}{4}+\frac{5}{4}-\frac{3\sqrt{5}}{2}$

= 7

77. Let $A=\begin{pmatrix}1 & 0 & 0\\ 0 & 4 & -1\\ 0 & 12 & -3\end{pmatrix}$. Then the sum of the diagonal elements of the matrix $(A + I)^{11}$ is equal to

(1) 6144 (2) 2050 (3) 4097 (4) 4094

Sol. **(3)**

$$A = \begin{bmatrix}1 & 0 & 0\\ 0 & 4 & -1\\ 0 & 12 & -3\end{bmatrix}$$

$$A^2 = \begin{bmatrix}1 & 0 & 0\\ 0 & 4 & -1\\ 0 & 12 & -3\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 4 & -1\\ 0 & 12 & -3\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & 4 & -1\\ 0 & 12 & -3\end{bmatrix}$$

$A^3 = A^4 = A^5 \ldots = A$

$(A + I)^{11} = {}^{11}C_0A^{11} + {}^{11}C_1A^{10} + {}^{11}C_2A^9 + \ldots {}^{11}C_{11}I$

$= =\left({}^{11}C_0 + {}^{11}C_1 + {}^{11}C_2 + \ldots {}^{11}C_{10}\right)A + I$

$= (2^{11} - 1)A + I$

= 2047 A + I

Sum of diagonal element = 2047(1 + 4– 3) +3

= 4097

78. The number of real roots of the equation $\sqrt{x^2-4x+3}+\sqrt{x^2-9}=\sqrt{4x^2-14x+6}$, is :

(1) 3 (2) 1 (3) 2 (4) 0

Sol. (2)

$$\sqrt{x^2-4x+3}+\sqrt{x^2-9}=\sqrt{4x^2-14x+6}$$

$$\sqrt{(x-3)(x-1)}+\sqrt{(x-3)(x+3)}=\sqrt{4x^2-12x-2x+6}$$

$$\Rightarrow\sqrt{x-3}\left(\sqrt{x-1}+\sqrt{x+3}\right)=\sqrt{4x(x-3)-2(x-3)}$$

$$\Rightarrow\sqrt{x-3}\left(\sqrt{x-1}+\sqrt{x+3}\right)=\sqrt{(4x-2)(x-3)}$$

$$\Rightarrow\sqrt{x-3}\left(\sqrt{x-1}+\sqrt{x+3}-\sqrt{2(2x-1)}\right)=0$$

$$\sqrt{x-3}=0 \text{ or } \sqrt{x-1}+\sqrt{x+3}-\sqrt{2(2x-1)}=0$$

$$x = 3 \text{ or } \sqrt{x-1}+\sqrt{x+3}=\sqrt{2(2x-1)}$$

$$x - 1 + x + 3 + 2\sqrt{(x-1)(x+3)} = 4x - 2$$

$$\Rightarrow 2\sqrt{(x-1)(x+3)}=2x-4$$

$$\Rightarrow (x-1)(x+3)=(x-2)^2$$

$$\Rightarrow x^2+2x-3=x^2+4-4x$$

$$\Rightarrow 6x=7$$

$$x=\frac{7}{6} \text{ (not possible)}$$

Number of real root = 1

79. If $\sin^{-1}\frac{\alpha}{17}+\cos^{-1}\frac{4}{5}-\tan^{-1}\frac{77}{36}=0, 0<\alpha<13$, then $\sin^{-1}(\sin\alpha)+\cos^{-1}(\cos\alpha)$ is equal to

(1) 16 (2) 0 (3) π (4) $16-5\pi$

Sol. (3)

$$\sin^{-1}\frac{\alpha}{17}+\cos^{-1}\frac{4}{5}-\tan^{-1}\frac{77}{36}=0,\ 0<\alpha<13$$

$$\sin^{-1}\frac{\alpha}{17}=\tan^{-1}\frac{77}{36}-\tan^{-1}\frac{3}{4}$$

$$=\tan^{-1}\left(\frac{\frac{77}{36}-\frac{3}{4}}{1+\frac{77}{36}\cdot\frac{3}{4}}\right)$$

$$\sin^{-1}\frac{\alpha}{17} = \tan^{-1}\left(\frac{8}{15}\right) = \sin^{-1}\left(\frac{8}{17}\right)$$

$$\frac{\alpha}{17} = \frac{8}{17}$$

$\alpha = 8$

$\sin^{-1}(\sin 8) + \cos^{-1}(\cos 8)$

$= 3\pi - 8 + 8 - 2\pi$

$= \pi$

80. Let $\alpha \in (0,1)$ and $\beta = \log_e(1-\alpha)$. Let $P_n(x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots + \frac{x^n}{n}, x \in (0,1)$.

Then the integral $\int_0^\alpha \frac{t^{50}}{1-t} dt$ is equal to

(1) $\beta + P_{50}(\alpha)$ (2) $P_{50}(\alpha) - \beta$ (3) $\beta - P_{50}(\alpha)$ (4) $-(\beta + P_{50}(\alpha))$

Sol. **4**

$\alpha \in (0,1), \beta = \log_e(1-\alpha)$

$P_n(x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \ldots\ldots + \frac{x^n}{n}$, $x \in (0, 1)$

$$\int_0^\alpha \frac{t^{50} - 1 + 1}{1-t} dt$$

$$-\int_0^\alpha \frac{1 - t^{50}}{1-t} dt + \int_0^\alpha \frac{1}{1-t} dt$$

$$-\int_0^\alpha \left(1 + t + t^2 + \ldots\ldots t^{49}\right) dt - \left[\ln(1-t)\right]_0^\alpha$$

$$-\left[t + \frac{t^2}{2} + \frac{t^3}{3} + \ldots\ldots \frac{t^{50}}{50}\right]_0^\alpha - \ln(1-\alpha)$$

$$-\left[\alpha + \frac{\alpha^2}{2} + \frac{\alpha^3}{3} + \ldots\ldots + \frac{\alpha^{50}}{50}\right] - \ln(1-\alpha)$$

$- P_{50}(\alpha) - \ln(1-\alpha)$

$- (\beta + P_{50}(\alpha))$

Section : Mathematics Section B

81. Let $\alpha > 0$, be the smallest number such that the expansion of $\left(x^{\frac{2}{3}} + \frac{2}{x^3}\right)^{30}$ has a term

$\beta x^{-\alpha}, \beta \in \mathbb{N}$. Then α is equal to

Sol. **2**

$$T_{r+1} = {}^{30}C_r\left(x^{\frac{2}{3}}\right)^{30-r}\left(\frac{2}{x^3}\right)^4$$

$$= {}^{30}C_r 2^r x^{\frac{60-11r}{3}}$$

$$\frac{60-11r}{3} < 0$$

11 r > 60

$r > \dfrac{60}{11}$

r = 6

$T_7 = {}^{30}C_6\ 2^6 x^{-2}$ then

$\beta = {}^{30}C_6 \times 2^6 \in N$

$\alpha = 2$

82. Let for $x \in \mathbb{R}$,

$$f(x) = \frac{x+|x|}{2} \text{ and } g(x) = \begin{cases} x, & x < 0 \\ x^2, & x \ge 0 \end{cases}.$$

Then area bounded by the curve $y = (f \circ g)(x)$ and the lines $y = 0, 2y - x = 15$ is equal to

Sol. **72**

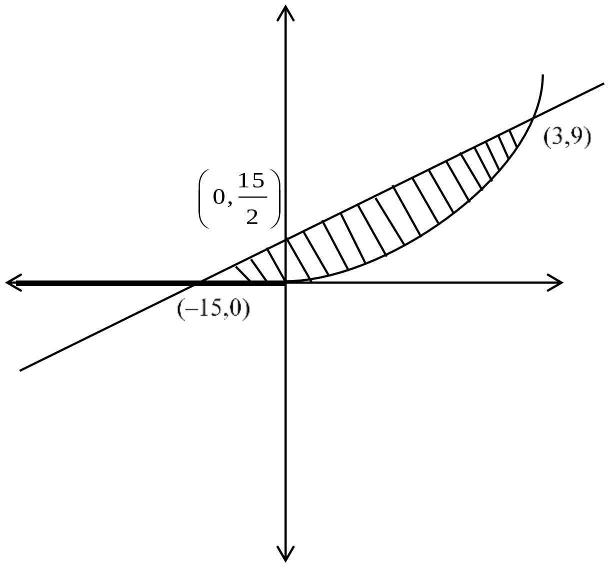
$$f(x) = \frac{x+|x|}{2} = \begin{cases} x & x \ge 0 \\ 0 & x < 0 \end{cases}$$

$$g(x) = \begin{cases} x^2 & x \ge 0 \\ x & x < 0 \end{cases}$$

$$\text{Fog}(x) = f\{g(x)\} = \begin{cases} g(x) & g(x) \ge 0 \\ 0 & g(x) < 0 \end{cases}$$

$$\text{fog}(x) = \begin{cases} x^2 & x \ge 0 \\ 0 & x < 0 \end{cases}$$

given lines are 2y – x = 15 and y = 0



Area = $\int_0^3 \left(\frac{x+15}{2} - x^2\right)dx + \frac{1}{2}\times\frac{15}{2}\times 15$

$= \left.\frac{x^2}{4} + \frac{15x}{2} - \frac{x^3}{3}\right]_0^3 + \frac{225}{4}$

$= \frac{9}{4} + \frac{45}{2} - 9 + \frac{225}{4}$

Area = 72

83. Number of 4-digit numbers that are less than or equal to 2800 and either divisible by 3 or by 11 , is equal to

Sol. **710**

4 digit number which are less then 2800 are 1000 – 2799

Number which are divisible by 3

$2799 = 1002 + (n - 1)\,3$

$n = 600$

Number which are divisible by 11 in 1000 – 2799

= (Number which are divisible by 11 in 1 – 2799)

– (Number which are divisible by 11 in 1 – 999)

$= \left[\frac{2799}{11}\right] - \left[\frac{999}{11}\right]$

= 254 – 90

= 164

Number which are divisible by 33 in 1000 – 2799

= (Number which are divisible by 33 in 1 – 2799) – (Number which are divisible by 33 in 1 – 999)

$= \left[\frac{2799}{33}\right] - \left[\frac{999}{33}\right]$

= 84 – 30 = 54

total number = n(3) + n(11) – n(33)

= 600 + 164 – 54 = 710

84. If the variance of the frequency distribution

x_i	2	3	4	5	6	7	8
Frequency f_i	3	6	16	α	9	5	6

is 3 , then α is equal to

Sol. **5**

x_i	f_i	$d_i = x_i - 5$	$(f_i\ d_i)^2$	$f_i d_i$
2	3	–3	27	–9
3	6	–2	24	–12
4	16	–1	16	–16
5	α	0	0	0
6	9	1	9	9
7	5	2	20	10
8	6	3	54	18

$$\sigma^2 = \frac{\sum f_i d_i^2}{\sum f_i} - \left(\frac{\sum f_i d_i}{\sum f_i}\right)^2$$

$$= \frac{150}{45+\alpha} - 0 = 3$$

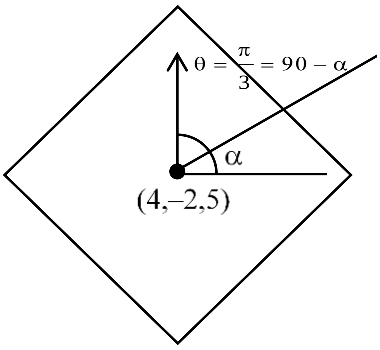
$$\Rightarrow 150 = 135 + 3\alpha$$

$$\Rightarrow 3\alpha = 15$$

$$\Rightarrow \alpha = 5$$

85. Let θ be the angle between the planes $P_1 : \vec{r} \cdot (\hat{\imath} + \hat{\jmath} + 2\hat{k}) = 9$ and $P_2 : \vec{r} \cdot (2\hat{\imath} - \hat{\jmath} + \hat{k}) = 15$. Let L be the line that meets P_2 at the point $(4, -2, 5)$ and makes an angle θ with the normal of P_2. If α is the angle between L and P_2, then $(\tan^2\theta)(\cot^2\alpha)$ is equal to

Sol. **9**



$$\cos\theta = \frac{\langle 1, 1, 2\rangle . \langle 2, -1, 1\rangle}{6}$$

$$= \frac{2-1+2}{6} = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

then $\frac{\pi}{2} - \alpha = \frac{\pi}{3}$

$$\alpha = \frac{\pi}{6}$$

$$\left(\tan^2\theta\right)\left(\cot^2\alpha\right) = \left(\sqrt{3}\right)^2 \times \left(\sqrt{3}\right)^2 = 9$$

86. Let 5 digit numbers be constructed using the digits 0,2,3,4,7,9 with repetition allowed, and are arranged in ascending order with serial numbers. Then the serial number of the number 42923 is

Sol. **2997**

2	$\frac{\ }{6}$	$\frac{\ }{6}$	$\frac{\ }{6}$	$\frac{\ }{6} = 1296$
3	$\frac{\ }{6}$	$\frac{\ }{6}$	$\frac{\ }{6}$	$\frac{\ }{6} = 1296$
4	0	$\frac{\ }{6}$	$\frac{\ }{6}$	$\frac{\ }{6} =216$
4	2	0	$\frac{\ }{6}$	$\frac{\ }{6} = 36$
4	2	2	$\frac{\ }{6}$	$\frac{\ }{6} = 36$
4	2	3	$\frac{\ }{6}$	$\frac{\ }{6} = 36$
4	2	4	$\frac{\ }{6}$	$\frac{\ }{6} = 36$
4	2	7	$\frac{\ }{6}$	$\frac{\ }{6} = 36$
4	2	9	0	$\frac{\ }{6} = 6$
4	2	9	2	$\underline{0} = 1$
4	2	9	2	$\underline{2} = 1$
4	2	9	2	$\underline{3} = \frac{1}{2997}$

87. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}| = \sqrt{14}, |\vec{b}| = \sqrt{6}$ and $|\vec{a} \times \vec{b}| = \sqrt{48}$.

Then $(\vec{a} \cdot \vec{b})^2$ is equal to

Sol. **36**

$$\left|\vec{a} \times \vec{b}\right|^2 = |\vec{a}|^2 \left|\vec{b}\right|^2 - \left(\vec{a} \cdot \vec{b}\right)^2$$

$$48 = 14 \times 6 - \left(\vec{a} \cdot \vec{b}\right)^2$$

$$\left(\vec{a} \cdot \vec{b}\right)^2 = 84 - 48$$

$$\left(\vec{a} \cdot \vec{b}\right)^2 = 36$$

88. Let the line $L: \frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{1}$ intersect the plane $2x + y + 3z = 16$ at the point P. Let the point Q be the foot of perpendicular from the point $R(1,-1,-3)$ on the line L. If α is the area of triangle PQR, then α^2 is equal to

Sol. **180**

Point on line L is $(2\lambda + 1, -\lambda - 1, \lambda + 3)$

If above point is intersection point of line L and plane then

$2(2\lambda + 1) + (-\lambda - 1) + 3(\lambda + 3) = 16$

$\lambda = 1$

Point $P = (3, -2, 4)$

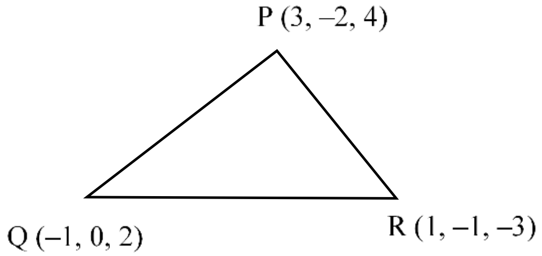
Dr of $QR = < 2\lambda, -\lambda, \lambda + 6 >$

Dr of $L = < 2, -1, 1 >$

$4\lambda + \lambda + \lambda + 6 = 0$

$\lambda = -1$

$Q = (-1, 0, 2)$



$\overrightarrow{QR} = 2\hat{i} - \hat{j} - 5\hat{k}$

$\overrightarrow{QP} = 4\hat{i} - 2\hat{j} + 2\hat{k}$

$$\overrightarrow{QR} \times \overrightarrow{QP} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & -5 \\ 4 & -2 & 2 \end{vmatrix} = -12\hat{i} - 24\hat{j}$$

$$\alpha = \frac{1}{2} \times \sqrt{144 + 576}$$

$$\alpha^2 = \frac{720}{4} = 180$$

$\alpha^2 = 180$

89. Let $a_1, a_2, \ldots, a_n$ be in A.P. If $a_5 = 2a_7$ and $a_{11} = 18$, then $12\left(\frac{1}{\sqrt{a_{10}}+\sqrt{a_{11}}} + \frac{1}{\sqrt{a_{11}}+\sqrt{a_{12}}} + \cdots + \frac{1}{\sqrt{a_{17}}+\sqrt{a_{18}}}\right)$ is equal to

Sol. **8**

Given that

$a_5 = 2a_7$

$a_1 + 4d = 2(a_1 + 6d)$

$a_1 + 8d = 0$

$a_1 + 10d = 18$

$a_1 = -72, d = 9$

$a_{18} = a_1 + 17d = -72 + 153 = 81$

$a_{10} = a_1 + 9d = 9$

$$12\left(\frac{\sqrt{a_{11}}-\sqrt{a_{10}}}{d}+\frac{\sqrt{a_{12}}-\sqrt{a_{11}}}{d}+......\frac{\sqrt{a_{18}}-\sqrt{a_{17}}}{d}\right)$$

$$= 12\left(\frac{\sqrt{a_{18}}-\sqrt{a_{10}}}{d}\right)$$

$$= \frac{12\times(9-3)}{9}=8$$

90. The remainder on dividing 5^{99} by 11 is :

Sol. **9**

$5^{99} = 5^4\, 5^{95}$

$= 625\ (5^5)^{19}$

$= 625\ (3125)^{19}$

$= 625(3124 + 1)^{19}$

$= 625(11\lambda + 1)$

$= 11\ \lambda \times 625 + 625$

$= 11\ \lambda \times 625 + 616 + 9$

$= 11 \times k + 9$

Remainder = 9